Nanophotonics

Class 1: September 24, 2010

Limit your answers to two pages and write clearly please.

Assignment 1:

We will derive an expression for the **optical constants of metals** in a similar way as we did for dielectrics. The optical response of metals is dominated by the free conduction electrons, which are not bound to nuclei. Accordingly, they do not experience a restoring force. However, damping is still important. For simplicity, we will neglect the contribution of bound electrons altogether.

a) Starting from the electron equation of motion in metals, and assuming that both **E** and **J** are harmonically oscillating with frequency ω, show that

$$\mathbf{J} = \frac{\sigma}{1 - i\omega/\gamma}\mathbf{E} \qquad \text{where} \qquad \sigma = \frac{Ne^2}{m\gamma}$$

b) Use the above expression to solve the wave equation, assuming $\mathbf{E} = \mathbf{E}_0 e^{i(k_z z - \omega t)}$, and derive the (complex) index of refraction.

Note: write the equation in terms of the plasma frequency, defined as $\omega_p^2 = \frac{Ne^2}{m\varepsilon_0}$.

c) Calculate the plasma frequency and corresponding wavelength of silver and aluminium.

d) Make a plot of the real and imaginary parts of ε as a function of frequency for silver and aluminium. Describe the characteristic trends.

e) In what frequency (wavelength) ranges are these metals
   a) transparent
   b) absorbing
   c) reflecting
Motivate your answers.

- **Please turn to the next page for assignment 2 -**

Assignment 2:

We will investigate the surface plasmon resonance frequency of small metal particles. When a small metal particle is placed in a homogeneous electric field, the negative charges are displaced from the positive ones. In the quasi-static limit, i.e. the case in which the particles are small compared to the wavelength, this response can be described by a dipole. The electric polarizability for an ellipsoid with axes a, b, and c, is then a diagonal matrix (in the principle axes system of the particle) with elements given by:

$$\alpha_i = 4\pi\varepsilon_0 (abc) \frac{\varepsilon - \varepsilon_m}{3\varepsilon_m + 3L_i(\varepsilon - \varepsilon_m)}$$

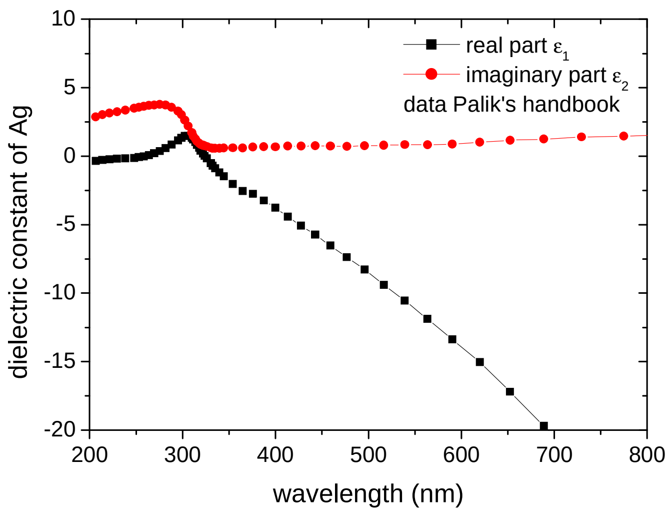
We observe that the polarizability is dependent on

- shape via the geometrical factors L_a, L_b and L_c (function of eccentricity). The sum of all L_i is 1.
- size via the length of the principle axes a, b, and c (volume)
- the metal via the dielectric constant ε
- the surrounding medium via the dielectric constant ε_m

A special case of an ellipsoid is the sphere, with all axes equal to the radius of the particle. For a sphere, the geometrical factor L_i is 1/3 and the polarizability is then isotropic (proportional to the identity matrix). The fact that the polarizability is a matrix is only useful for question e).

a) What is the resonance condition for a small metal sphere placed in an electric field?

b) Explain why the color of glass containing small Ag particles is yellow (figure right). Use the dielectric constants of Ag given in the left graph (and **not** a Drude model applied to silver). The refractive index of glass is 1.5.



- **assignment continues on page 3 -**

For prolate spheroids (long axis a, two short axes b=c), the geometrical factor L_a can be expressed as a function of the eccentricity *e* by

$$L_a = \frac{1-e^2}{e^2}\left[\frac{1}{2e}\ln\left(\frac{1+e}{1-e}\right)-1\right]$$

With

$$e^2 = 1-(b/a)^2$$

We see that the resonance conditions for prolate spheroids are different than for an Ag sphere and vary with the eccentricity. This tunability is the main advantage of non-spherical particles. In particular, they are used to enhance the excitation efficiency of dye molecules.

(c) For a molecule absorbing at 450nm, what aspect ratio of the particle should be approximately chosen if the surrounding medium is glass? (hint: you may want to plot $L_a(e)$ using a computer)

(d) In an experiment, we want to resonantly excite the particles with a HeNe laser. To achieve resonance, we will change the surrounding medium. Assuming a spherical particle, what should the index of the medium be for this experiment?

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Return your homework on Friday, October 1st. No late returns!