Nanophotonics

Class 2: October 1, 2010

Assignment

1.

- Derive the dispersion relation for surface plasmon polaritons (SPPs).
  - Use Maxwell's equations and the electromagnetic wave equation together with the ansatz for SPPs:
    The SPPs propagate in the positive *x* direction, and their fields can be written as:
    $z > 0$ (the dielectric (*d*) with dielectric constant ε_d)

$$H_{d,y}(x,z,t) = H_{yd} e^{i(k_x x + k_{z,d} z - \omega t)}$$

$$E_{d,x}(x,z,t) = E_{xd} e^{i(k_x x + k_{z,d} z - \omega t)}$$

$$E_{d,z}(x,z,t) = E_{zd} e^{i(k_x x + k_{z,d} z - \omega t)}$$

    $z < 0$ (the metal (*m*) with dielectric constant ε_m)

$$H_{m,y}(x,z,t) = H_{ym} e^{i(k_x x - k_{z,m} z - \omega t)}$$

$$E_{m,x}(x,z,t) = E_{xm} e^{i(k_x x - k_{z,m} z - \omega t)}$$

$$E_{m,z}(x,z,t) = E_{zm} e^{i(k_x x - k_{z,m} z - \omega t)}$$

    where H_{yd} etc. are constants. The field components not specified above are 0, since SPPs are 'transverse magnetic' (TM) waves.
  - Imply boundary conditions at the interface between ε_d and ε_m.
  - Assume non-magnetic material, so $\mu_d = \mu_m = 1$

Note:
For exercise 2 and 3 you have to compute complex roots using e.g. MatLab or Mathematica. If you do not have these programs available use the following approximations for the SPP wavevector:

$$k_x' \cong \frac{\omega}{c}\left(\frac{\varepsilon_1'\varepsilon_2}{\varepsilon_1'+\varepsilon_2}\right)^{1/2}$$

$$k_x'' \cong \frac{\omega}{c}\left(\frac{\varepsilon_1'\varepsilon_2}{\varepsilon_1'+\varepsilon_2}\right)^{3/2} \frac{\varepsilon_1''^2}{2\varepsilon_1'^2}$$

2.

- Plot the dispersion relation and decay length (propagation length) of SPPs at a Au/air interface.
  - use the modified Drude model for the dielectric constant of Au.
  - use as plasma frequency $\omega_p = 1.40 \times 10^{16}$ **rad s^{-1}**, collision frequency $\gamma = 1.27 \times 10^{14}$ **s^{-1}** and $\varepsilon_{inf} = 6.13$

- Identify the different frequency regions of the SPP dispersion.

3.

- Calculate the SPP wavelength for an Au/air interface for photon energies of 0.5 eV and 2.8 eV and compare the results to the free space wavelength

- Calculate the SPP decay length for the given energies and comment on the results

- In exercise 1 you derived a relation for the wavevectors along and perpendicular to the surface. How does this in general relate the vertical (transverse) confinement of the SPPs to the SPP wavelength? Illustrate this quantitatively for the given energies.

- Compare the vertical confinement in the dielectric medium with the vertical confinement in the metal. Comment on the nature of the calculated differences

- How does the SPP wavelength and decay change when we consider an Au/Al_2O_3 interface?
    - use a refractive index of Al_2O_3 *n=1.75*

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