Answered questions for Assignment #4
(Please submit questions in English)

1. I've got a small question about the assignment: in #1B there are terms with a "hat". Does this mean unity vector? So for instance "r hat" would point in the direction of position and have the absolute value 1.

Yes. For details see the entry on the “Unit Vector” at Wikipedia.

2. What does R_0 mean in 1B?

Please see the final question in this document.

3. For problem 1B: how do you write equation 1 in terms of cosines?

I am not sure that Google translated your question correctly... I am not sure which cosines you are thinking about – note that the hints in 1C are not used in 1B. If you are confused about the complex number representation please review the Euler Identity on Wikipedia.

4. Probably enough to have a half circle to integrate it?

You need to integrate over an enclosing volume. As written, Eq. 4 suggests you integrate over all 4π surface angles. If you spot a symmetry that makes the integral easier for you feel free to apply it.

5. I have a few questions about the assignment for nanophotonics. In 1C an expression is given for rd in case R<<d, but shouldn't it be R>>d?

Yes, this is a typo.

6. In 1Cd, for the part of the integral that depends on theta I found the expression sin(theta)^3*cos(2kd cos(theta)), is that correct?

Being correct is less important than understanding why this may or may not be correct. This isn’t a question that I can answer in a useful way.

7. In 1Cd I have to plot the power normalized to P_{free}. I assume that you have to plot it as a function of *d*? But how can you do that with mathematica, as Nintegrate can only calculate the numerical value of the integral?

The intent was that you plot P/P_{free} versus a dimensionless normalized quantity (perhaps *kd*?). However if you can think up an alternative way to plot your result, that is also okay. To use Nintegrate, you should make a change of variables and express the integrand in terms of your dimensionless normalized parameter.

8. In the hint for 1Cb, is R <<d a mistake?

Yes, this is a typo. However, I would encourage you to think about what parts of the problem would change if we instead made our calculation in the extreme near field (R<<d). For example, do you think the radiated power would be different?

9. Is it the intention to question 1C to calculate the same as in 1B, but with the complete expressions for **E** and **B** instead of the far-field approaches that we have first made?

You can make the calculation in the far-field provided that you understand the approximations you are making.

10. In question 1A it's suggested that the dipole moment *p* can be calculated by introducing an image dipole at a distance *d* at the other side from the mirror. The dipole moment is dependent on the charge of the dipole on the right and the distance to the mirror. What is the situation? Do we have a charge or a dipole on the right of the mirror?

You may wish to review the material on dipoles at Wikipedia; here we suggest that you imagine the dipole as two charges separated by an infinitesimal distance and separately apply the method of images to each imaginary charge in the dipole.

11. In 1B, we have to calculate P_{free} which depends on *p*. So we have to "choose?" a value for the distance? On top of that the output power P_{free} will increase if the distance with respect to the mirror is increased? That seems wrong.

The dipole moment *p* is not defined by the distance to the mirror, *d*. There is no mirror in 1B; you are calculating the power radiated by the dipole in the absence of the mirror.

12. I am now working on problem 1C and I have reached part d where I have to integrate using Mathematica. However I found that the integrand still contains the term in time (t) in the complex phase factor. This leads me to several questions: How do I get rid of this t term? Is it safe to take the average in time by introducing a factor of 1/2? Or do i have to assume that t = 0 when I integrate? What is the reasonable value for R that I should choose? Is it okay to take any value of R as long as R >> Ro(k) and R >> d?

Note that the time averaged pointing vector is proportional to $\mathbf{E}\text{x}\mathbf{B}^*$ - when you take the complex conjugate of the magnetic field, the time dependent terms will drop out of the expression. Likewise, if you include the factor (r^2) from the Jacobian in the integral over the surface of the sphere the *R* terms will drop out of the integrand.

13. I assumed that, whereas in equation (3) the Poynting vector is a vector, in equation (4) the length of the vector (a number) is meant. Is this correct?

The Poynting vector is a vector in the direction of $\hat{r}$. When you evaluate the flux through the surface of the sphere, you take the dot product of the Poynting vector with the surface normal. In equation (4) this results in you taking the length of the vector. However, note that you could also calculate the power by integrating over the six sides of an enclosing box – in this case, you would not be evaluating the length of the vector.

14. In exercise 1Cb I assumed that we should take R>>d instead of R<<d. Is this correct?

You are correct; this was a typo.

15. In exercise 2A, both plots are different in the new version of the assignment. I assume that the LDOS plot is still the LDOS normalized to the case without mirror. However, the LDOS appears to no longer approach 1 for large distance from the mirror. I think something is wrong there. In addition, in the exercise it says that the ions are in a medium of index 1.5 while in the picture it says dielectric constant = 2.5.

The LDOS is normalized to the density of states in a uniform medium with ε=2.5. For very large distances from the mirror, the dipole is embedded 1.6 nm below the surface of an infinite half space with ε=2.5. The other infinite half-space is taken to be vacuum (ε=1). The LDOS in a uniform medium scales with the index of refraction, so the normalizing density of states is ~1.6*LDOS(n=1). Because we are next to a region with a lower index the LDOS is reduced from this value in our geometry when we are far from the mirror. The actual limiting value is ~0.717*LDOS(n=1.6) or 1.133*LDOS(n=1).

The index in the text of the problem was left over from last year's exercise while the epsilon value in the picture was updated from Amos and Barnes paper. I have changed the assignment so that these numbers now agree, but the exact value is not important for the problem. You just need to convey that you understand the physics of why the LDOS oscillates and get an approximately correct estimate for the wavelength.

16. I just started working on assignment 4 and I got a bit confused with problem 1B. What does R_0 in that problem refer to? And which relation or equation should I use in order to get an expression for $R_0(k)$?

Here I would like you to think about what it means to be in the "far-field": the region in which the 1/r terms of the field are much larger than the $1/(r^2)$ terms. The problem asks you to find an expression for a distance "R_0" such that the far-field region is defined by the inequality "$r \gg R_0$." It might help you to start from the definition of the far-field:

$$\left|\left(\frac{1}{r}\text{ terms}\right)\right| \gg \left|\left(\frac{1}{r^2}\text{ terms}\right)\right|$$

Note that this is an inequality in *magnitudes*. If you write down this expression for either the **E** or the **B** field and do some algebra, you will arrive at a single controlling expression of the form:

$$r \gg f(k)$$

In the problem I have suggested that you call this function $R_0(k)$.