Nanophotonics
Class 9: December 17th, 2010
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Assignment

Exercise 1
Ray tracing of a perfect lens

First we consider a normal lens, governed by the lens formula:

$$\frac{1}{v}+\frac{1}{b}=\frac{1}{f}$$

that relates the distance v of object to lens to the distance b of image to lens via the focal length f of the lens.

Suppose we make an image of an object in an object plane at distance v of the lens using a positive lens of focal distance f.

(a) What is the orientation of the image in the image plane? What happens if the object distance is less than f?

(b) Make a plot of the magnification of the object versus object distance v/f (in units of f), and of the total distance $b+v$ between object and image versus v.

Now we consider a slab of material with refractive index $n=-1$ and thickness d. Using ray tracing based on Snell's law

(c) What is the magnification and orientation of the image if the object is at a distance $d/2$ of the slab surface? Show a diagram that traces the rays from object to image. Notice the virtual image inside the slab.

(d) What happens with the magnification and object-to-image distance when the object is brought closer to the lens? Is an image still formed if the object is moved further away from the lens?

Exercise 2
Magnetic-atom design

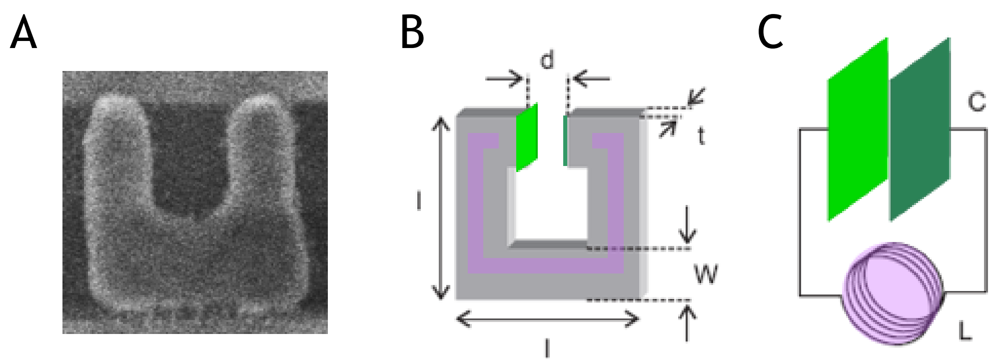

Figure A shows a scanning electron microscope (SEM) image of a split ring resonator (SRR) made of Au on a SiO_2 substrate by electron beam lithography technique. The approximate dimensions of the SRR, as depicted in Figure B, are *l*=200 nm, *w*=80 nm, *t*=30 nm, and *d*=80 nm, where *l*=length, *w*=width, *t*=thickness and *d*=gap size of the SRR. An SRR can be described as an LC-circuit with a parallel plate capacitance C and inductance L of an N-loop coil, as depicted in Figure C. These parameters depend on the SRR geometry. Using 1st year electrodynamics (see University Physics book) and the illustration given in Figure B, assuming that $l = 1, w = 0.25l, t = 0.2l, d = 0.3l, \varepsilon_c = 1$, calculate

a) Capacitance C
b) Inductance L
c) Draw an equivalent circuit including Ohmic loss
d) The Ohmic resistance R can be estimated using $\varepsilon = 1 + \frac{i\sigma}{\omega}$, where σ is the conductivity and $\rho = \sigma^{-1}$. From the above expression you get $\rho = \frac{1}{\omega\varepsilon_0 \operatorname{Im}\varepsilon} \sim 0.5 \cdot 10^{-5}$. Approximate the SRR as a wire of cross-section $A = wt$, length $l_{eff} = 4(l - w) - d$ and resistance $R = \rho \frac{l_{eff}}{A}$. Calculate the impedance Z of the LRC circuit. How does the resonance ω scale with size *l* taking *R=0*? For resonance at $\lambda = 1.5\mu m$, how small should l be, assuming the parameters given above?

As you know, the response of a circuit to an applied voltage is proportional to $\frac{1}{Z}$. In the presence of loss *R*, we can write $\frac{1}{Z}$ in the form of $\frac{1}{Z} = \frac{O}{(\omega^2 - \omega_0^2) + i\Gamma^2}$, with ω_0 the resonance frequency of the circuit without *R*, $O = \omega_0^2 \omega C$ and $\Gamma^2 = \omega_0^2 \omega RC$. The linewidth of the resonance is defined by the full width half maximum (FWHM) of the resonance peak. Calculate the FWHM of $\frac{1}{Z}$. What is the effect of the size scaling on the inverse linewidth Q?

f) We wish to estimate the effective magnetic permeability μ_{eff} of a lattice of SRRs.

We start by finding the circulating current I in the SRR. An external magnetic field $\vec{H} = H_0 e^{-i\omega t}$ drives a current I in the ring, which is related through Ohm's law $V = I \cdot Z$ to the "electromotive" potential V. The electromotive V results from Faraday's law of induction $V = -\oint \frac{d\vec{H}}{dt} \mu_0 dA = i\omega \vec{H} \mu_0 l^2$. Now, the induced current gives rise to individual magnetic dipole moments $\vec{m} = I \cdot l^2 = \alpha \cdot \vec{H}$ that contribute to the microscopic magnetic field $B_{eff} = \mu_0(\vec{H} + \vec{M}) = \mu_0(\vec{H} + \frac{N}{V}\vec{m}) = \mu_0 \mu_{eff} \vec{H}$, where $\frac{N}{V}$ is the magnetic dipole moment number density with N individual magnetic dipole moments per volume V. Find the magnetic polarizability α, the expression for the magnetic dipole moment $\vec{m}$ and proceed to obtain the expression for the effective magnetic permeability μ from the magnetic induction B. Fill in numbers of your choice for SRRs with l=200 nm with a cubic lattice constant a=300 nm and plot the magnetic permeability versus size.