Erbium-implanted high-\(Q\) silica toroidal microcavity laser on a silicon chip

Bumki Min, Tobias J. Kippenberg, Lan Yang, and Kerry J. Vahala
Thomas J. Watson Laboratory of Applied Physics, California Institute of Technology, Pasadena, California 91125, USA

Jeroen Kalkman and Albert Polman
FOM Institute for Atomic and Molecular Physics, Kruislaan 407, 1098 SJ Amsterdam, The Netherlands
(Received 17 March 2004; published 16 September 2004)

Lasing from an erbium-doped high-\(Q\) silica microcavity coupled to a tapered optical fiber is demonstrated and analyzed. Average erbium ion concentrations were in the range 0.009–0.09 at. %, and a threshold power as low as 4.5 \(\mu\)W and an output lasing power as high as 39.4 \(\mu\)W are obtained from toroidal cavities with major diameters in the range 25–80 \(\mu\)m. Controlling lasing wavelength in a discrete way at each whispering-gallery mode was possible by changing the cavity loading, i.e., the distance between the tapered optical fiber and the microcavity. Analytic formulas predicting threshold power and differential slope efficiency are derived and their dependence on cavity loading, erbium ion concentration, and \(Q\) factor is analyzed. It is shown that the experimental results are in good agreement with the derived formulas.

DOI: 10.1103/PhysRevA.70.033803  PACS number(s): 42.55.Sa, 42.60.Da, 42.81.Qb, 42.82.—m

I. INTRODUCTION

Ultrahigh-quality-factor (\(Q > 10^8\)), dielectric microcavity resonators possessing whispering-gallery modes (WGMs) have been investigated intensively for the last decade. Cavity quality factors (\(Q\) factors) as high as \(8 \times 10^9\) [1,2], along with small mode volumes, have been essential driving forces for fundamental research areas such as cavity quantum electrodynamics, nonlinear optics, biosensing, and microscale lasers [1–17]. The first studied ultrahigh-\(Q\) optical microcavities were liquid microdroplets. In the pioneering work of Chang and Campillo, nonlinear optical effects were investigated in such structures illuminated by free-space beams [3–6]. However, microdroplets are inherently transient and the free-space excitation of resonances was inefficient.

Two kinds of “solid” microcavities, which are capable of achieving ultrahigh-\(Q\) performance, have been developed to overcome these stability and efficiency issues related to liquid droplets. The older of these is a spherical “droplet” of silica which exhibits \(Q\) factors of up to \(8 \times 10^9\) in the near infrared [1,2]. These solid microspheres, like their liquid counterparts, rely upon surface tension to create an exquisitely smooth dielectric boundary [2] along which the whispering-gallery modes are confined. More recently, a surface-tension-induced structure in the form of a toroidal microcavity has enabled access to \(Q\) factors in excess of \(10^8\), the highest value reported on a planar substrate [18]. These toroidal microcavities are silica based and fabricated on silicon chips combining standard silicon technology with ultrahigh-\(Q\) fabrication techniques. This synergistic approach allows creation of dense arrays of ultrahigh-\(Q\) microcavity structures in parallel on a silicon wafer, allowing the integration with complementary electrical, optical, or mechanical functionality. In addition, lithographic methods can be used to control size, rendering the devices highly reproducible.

Among the practical applications of ultrahigh-\(Q\) microcavities, rare-earth doped microcavity lasers have been successfully demonstrated in recent years. These include whispering-gallery-type microsphere lasers doped with neodymium, erbium, and ytterbium-sensitized erbium [8–15]. For the case of a neodymium-doped microsphere laser demonstrated by Sandoghdar et al. [8], the (absorbed) threshold power was as low as 200 nW. Using the chip-based toroidal microcavities described above, a sol-gel coating technique was applied recently to fabricate erbium-doped toroidal microcavity lasers on a silicon chip [16]. In addition to the advantages in fabrication, these devices provide improved spectral performance as compared to microspheres owing to their reduced modal spectra [16,18]. In particular, single-mode operation was possible in these devices. As a complementary method of gain functionalization that is more compatible with standard VLSI processing, we have recently demonstrated erbium-implantation-based toroidal microcavity lasers [17]. For both types of erbium-doped resonators (made by sol-gel processing or implantation), the internal dynamics and the dependence on cavity loading between the waveguide (tapered optical fiber) and the microcavity have not been thoroughly investigated.

In this paper, we used the ion implantation method to incorporate erbium ions into ultrahigh-\(Q\) toroidal microcavities and study in detail various lasing characteristics when these devices are coupled to a tapered optical fiber for excitation. In addition, experimental results are correlated with modeling of devices to understand the limits of performance as well as the basic lasing mechanisms. Using samples of varying sizes and erbium ion concentrations, the minimum threshold power was shown to be as low as 4.5 \(\mu\)W and the highest output power was measured to be 39.4 \(\mu\)W. A maximum unidirectional slope efficiency of 5.5% was observed. Lasing wavelength control between adjacent whispering-gallery modes was demonstrated and explained by a cavity-loading-induced change in the upper-state population of erbium ions.

The remainder of this paper is organized as follows. In Sec. II, we present a detailed description of erbium-doped high-\(Q\) toroidal microcavity lasers using coupled-mode formalism. In Sec. III, we briefly illustrate the fabrication pro-
cess used to create the erbium-implanted toroidal microcavities. In Sec. IV, we investigate experimentally the performance of erbium-implanted toroidal microcavity lasers and compare these experimental results with the proposed analysis. Finally, in Sec. V, we conclude and discuss future research directions.

II. ANALYSIS OF ERBIUM-DOPED HIGH-Q TOROIDAL MICROCAVITY LASERS

The analysis is based on the coupled-mode formalism for the resonator coupled to an external waveguide [19–23] with the addition of gain (or loss) terms from the excited erbium ions. Considering two resonant modes at the pump and (lasing) signal wavelengths in the microcavity, along with the pump wave coupled into the resonator from the waveguide (Fig. 1 shows the graphical rendering of the toroidal microcavity-tapered fiber system), the basic coupled equations of motion for the pump and signal field can be written as follows:

\[
\frac{da_s}{dt} = -\frac{1}{2\tau_s}a_s + g_sa_s,
\]

\[
\frac{da_p}{dt} = -\frac{1}{2\tau_p}a_p - l_p a_p + i\kappa_p s_p,
\]

where \(a_s, a_p\) are internal cavity fields at the signal and pump wavelengths (\(|a_s|^2\) is normalized to the energy in the cavity; and counter-rotating field coupling induced by surface inhomogeneity is not included in this analysis), \(\tau_s, \tau_p\) are cavity photon lifetimes at signal and pump wavelengths (which include passive cavity loss mechanisms such as surface scattering, absorption, and waveguide coupling [1,2,22] but exclude the gain/loss caused by the erbium ions), \(g_s, l_p\) are intracavity gain (or loss) coefficients depending on the mode, \(\kappa_p\) is the amplitude coupling coefficient between the resonator and the waveguide pump wave (which is defined as \(\sqrt{1/\tau_{ext}}\) and can be controlled by changing the gap width between the resonator and the waveguide), and \(s_p\) is the pump field amplitude in the input waveguide (\(|s_p|^2\) is normalized to the pump power in the waveguide). For simplicity, we have used the slowly varying envelope approximation (SVEA). Competing effects such as cooperative upconversion of erbium ions, excited-state absorption (pump or signal), pair-induced quenching [24–26], and the inclusion of spontaneous emission are not considered here. However, these effects can easily be incorporated analytically into the above basic equations by including appropriate additional rate terms and their associated rate equations. Mode splitting due to coupling of counter-rotating waves (induced by inhomogeneity of cavity surfaces) [21,27] is not considered.

Under steady-state conditions, the time derivative terms in (1) and (2) can be set to vanish, leading to the following two equations (3) and (4). The first equation is equivalent to the well-known threshold condition (i.e., cavity gain should be equal to cavity loss) for lasing, and the second equation relates the internal cavity energy at the pump wavelength to the input pump power at the waveguide-resonator junction

\[
g_s = \frac{1}{2\tau_s},
\]

\[
|a_p|^2 = \frac{\kappa_p^2}{\left(\frac{1}{2\tau_p} + l_p\right)}|s_p|^2.
\]

Even though these equations seem to be decoupled from each other, they are in fact coupled since the erbium upper-state population \(N_2\) determines both \(g_s\) and \(l_p\) as shown below [Eqs. (5) and (6)]. The intracavity gain and loss coefficients, \(g_s, l_p\), can be expressed as

\[
g_s = \frac{c}{2n_s} \left[ \frac{N_2}{N_T} (\alpha_s + g_s^*) - \alpha_s \right],
\]

\[
l_p = \frac{c}{2n_p} \left[ \frac{N_2}{N_T} (\alpha_p + g_p^*) + \alpha_p \right],
\]

where \(c\) is the speed of light, \(n_s, n_p\) are the refractive indices at the signal and pump wavelengths, respectively, \(N_T\) is the average erbium ion concentration, and \(\alpha_s, \alpha_p, g_{s,p}^*\) are Giles parameters [24,25,28] originally introduced for the modeling of erbium-doped fiber amplifiers and defined as

\[
\alpha_{s,p} = \Gamma_{s,p} N_T \sigma_{g,s,p},
\]

\[
g_{s,p}^* = \Gamma_{s,p} N_T \sigma_{e,s,p}.
\]

In these expressions \(\Gamma_{s,p}\) is an overlap factor (defined as the dimensionless spatial overlap integral between the normalized optical intensity distribution and the normalized erbium ion distribution) at signal and pump wavelengths and \(\sigma_{g,e,s,p}\) is the absorption/emission cross section of erbium ions at signal and pump wavelengths.

FIG. 1. Graphical rendering of the toroidal microcavity-tapered optical fiber system. Input/output waveguide (tapered optical fiber) and erbium-doped toroidal microcavity are shown along with an input pump wave \(s_p\) and internal cavity fields \(a_p\) and \(a_s\).
To fully utilize the simplicity of Giles parameters as given by (7) and (8) for further discussion, it is necessary to specify the overlap factors in the toroidal microcavities. However, to date, there has not been a detailed analytic model for the field distribution in the dielectric toroidal resonators. In addition, the erbium ion diffusion process during the laser reflow process (see Armani et al. [18], and also Sec. III) is not well known. Therefore, throughout this paper, the erbium ions will be assumed to diffuse homogeneously and the overlap factors of unity ($\Gamma_{s,p} = 1$) for both signal and pump wavelength will be assumed for simplicity (on the assumption that the mode field distribution is well confined in the microcavity). We note that the pump wavelength used is in the range 1460–1480 nm and therefore signal and pump overlap factors will be nearly the same.

### A. Upper-state population of erbium ions in microcavity

Combining Eq. (3) with (5), the normalized erbium upper-state population $N_2/N_T$ can be expressed as follows:

$$\frac{N_2}{N_T} = \frac{\alpha_s + \alpha_s^{\text{passive}}}{\alpha_s + g_s^{\text{passive}}},$$  

where the passive cavity signal loss, $\alpha_s^{\text{passive}}$, is defined as

$$\alpha_s^{\text{passive}} = \frac{n_s}{c\tau_s} = \frac{2\pi n_s}{\lambda_s Q_{T,s}^{\text{passive}}},$$

with $Q_{T,s}^{\text{passive}}$ the loaded quality factor at the signal wavelength, excluding the effects of erbium ions. Equation (9) is especially useful because it provides a simple relation between the normalized upper-state population in the microcavity and the parameters of microcavity and erbium ions. It reflects the well-known “clamping” of the population inversion under lasing conditions. The normalized upper-state population $N_2/N_T$ is shown in Fig. 2 as a function of gap width (cavity loading) and signal intrinsic $Q$ factor. The calculation is performed assuming $\Gamma_{s,p} = 1$, a signal wavelength of 1552.9 nm, erbium ion concentration of $2 \times 10^{19}$ cm$^{-3}$, and a cavity diameter of 50 μm coupled to a phase-matched tapered fiber. In the highly undercoupled regime (large gap width between the microcavity and the tapered fiber), the normalized erbium upper-state population is determined by intrinsic signal quality factor and Giles parameters. For ultrahigh-$Q$ factor, it is set near $\alpha_s/(\alpha_s + g_s^{\text{passive}}) = 0.39$ (determined by Giles parameters and therefore varying with lasing wavelength). At each preassumed lasing wavelength, a possible minimum $N_2/N_T$ is defined by $\alpha_s/(\alpha_s + g_s^{\text{passive}})$, below which lasing will not be permitted. As the cavity loading increases (gap width decreases), the loaded signal quality factor decreases and therefore the normalized erbium upper-state population increases (favoring the possibility of shorter wavelength lasing). It is well known that the peak gain wavelength [29] and the corresponding lasing wavelength are determined by the normalized erbium upper-state population. In Sec. IV, lasing wavelength control by change of cavity loading will be further examined on the basis of Eq. (9).

### B. $Q$ factors at pump and signal wavelengths

In Eq. (9) for the population inversion, cavity loss $\alpha_s^{\text{passive}}$ at the signal wavelength is the only variable that can be controlled (after fabrication) by changing the waveguide loading of the cavity. This passive cavity signal loss $\alpha_s^{\text{passive}}$ can be expressed using the loaded $Q$ factor as in (10). In the case of weak coupling between the resonator and the waveguide, the decomposition of the loaded $Q$ factor for the passive (i.e., without the effects of erbium ions) microcavity is given by the following simple form:

$$\frac{1}{Q_T^{\text{passive}}} = \frac{1}{Q_{S}^{\text{passive}}} + \frac{1}{Q_{\text{ext}}},$$

where $Q_{\text{ext}}$ is the external $Q$ factor of the microcavity (determined by cavity loading) and $Q_{S}^{\text{passive}}$ is the intrinsic $Q$ factor originating from loss mechanisms such as cavity surface scattering, adsorbed surface water, residual OH$^-$ impurities, whispering-gallery leakage, and bulk material absorption loss [1,2]. For undoped silica microcavities having diameters larger than approximately 20 μm and for the wavelength band of interest in this work (1400–1600 nm), intrinsic $Q$ factors are limited primarily by water adsorption and surface scattering [2,30]. It should also be noted that in the present work, the erbium implantation process could potentially alter $Q_{T,s}^{\text{passive}}$ by the introduction of absorbing (point) defects in the ion beam modified silica network structure [31].

Transitions between the ground- and excited-state manifolds ($^4I_{13/2}$ and $^4I_{15/2}$) of erbium introduce signal gain and pump absorption terms as described in (1) and (2). By extending the definition of $Q$ factor to include these effects, the following “erbium-population-inversion dependent” cavity $Q$ factors can be obtained:

$$\frac{1}{Q_T} = \frac{\lambda_s}{2\pi n_s} \left[ -\frac{N_2}{N_T} (\alpha_s + g_s^{\text{passive}}) + \alpha_s \right] = -\frac{\lambda_s Q_{T,s}^{\text{passive}}}{2\pi n_s} = -\frac{1}{Q_{T,s}^{\text{passive}}},$$

Fig. 2. (Color) Normalized upper-state population of erbium ions $N_2/N_T$ under condition (3) calculated as a function of gap width (cavity loading) and signal intrinsic $Q$ factor.
 terms of the internal cavity fields. From the rate equation for an excited population of erbium ions under steady-state conditions \([24,25,28]\), the following expression results:

\[
\frac{N_s}{N_T} = \frac{\alpha_p |a_s|^2 + |a_p|^2}{\alpha_s + g_s' |a_s|^2 + \alpha_p + g_p' |a_p|^2 + hN_T c \tau_{Er,N_T}},
\]

where \(V_m^p\) are the effective pump and signal mode volumes in the microfiber, \(h\) is the Planck’s constant, and \(\tau_{Er,N_T}\) is the lifetime of the erbium ions.

This normalized upper-state population is equated to its threshold value (assuming lasing condition and population clamping) from Eq. (9). After simple algebraic manipulation, the following relation between internal circulating pump and signal energies can be obtained:

\[
|a_s|^2 = -\frac{\nu_p n_p V_m^p}{\nu_s n_s V_m^s} \frac{\alpha_s |a_s|^2 + \alpha_p |a_p|^2}{\alpha_s + g_s' |a_s|^2 + \alpha_p + g_p' |a_p|^2} |a_p|^2
\]

\[
- \frac{N_s h}{\nu_p n_p V_m^p} \frac{\alpha_{Er} (\alpha_s + g_s^\text{passive})}{\alpha_s^\text{passive} (\alpha_s + g_s')} |a_s|^2,
\]

where \(\alpha_{Er} = n_s/c \tau_{Er,N_T}\). To relate these internal field quantities to the fiber-taper coupled quantities, the internal pump energy \(|a_p|^2\) is related to the input waveguide power \(|s_p|^2\) through the following expression [obtained by combining (4), (6), and (9)]:

\[
|a_p|^2 = \frac{4 n_p^2 \kappa_s^2 (\alpha_s + g_s')^2}{c^2 [(\alpha_s + g_s^\text{passive}) (\alpha_s + g_s') - (\alpha_s + g_s') (\alpha_s + g_s^\text{passive})]^2} \times |s_p|^2.
\]

Expressions (19) and (20) account for both waveguide loading and population-dependent coupling effects. To convert from internal cavity energy to output lasing power, the internal energy expression (19) is multiplied by \(\kappa_s^2 (P_L = \kappa_s^2 \times |a_s|^2)\).

Figure 3 shows the output lasing power as a function of input pump power for different cavity loading conditions. Input parameters are the same as for the calculations in Fig. 2. In addition, a pump wavelength of 1471 nm and \(Q_{s,p}^\text{passive} = 4 \times 10^7\) are used. As can be seen, the lasing threshold is a sensitive function of the gap width. The slope efficiency, defined as the differential output power versus input power, is also strongly dependent on the coupling conditions. In the next section, we will further discuss the threshold power and differential slope efficiency.

D. Threshold and slope efficiency

The input (fiber-launched) threshold pump power can be obtained directly from Eqs. (19) and (20) by setting the internal circulating signal energy to zero and solving for the required pump power.
\[ |s|_{th}^2 = N_p h \nu_m V_m \left( \frac{\nu_p h \nu_m}{\nu_p h \nu_m} \right) \left( \frac{c^2}{4n_p^2 \alpha_p} \right) \times \left[ (\alpha_p + \alpha_p^{\text{passive}})(\alpha_s + g_s) - (\alpha_p + g_p^{\text{passive}})(\alpha_s + \alpha_s^{\text{passive}}) \right]^2 \times \frac{\alpha_E (\alpha_s + \alpha_s^{\text{passive}})}{\alpha_p (\alpha_s + g_s) - (\alpha_p + g_p^{\text{passive}})(\alpha_s + \alpha_s^{\text{passive}})}. \] (21)

It should be noted that the threshold power derived here is a lower limit, as modal coupling of counter-rotating pump waves \([21,27]\), concentration-related quenching, and excited state absorption are not taken into account.

To obtain an expression for the coupled threshold power rather than input threshold power, it is necessary to account for the power coupled from the tapered optical fiber into the cavity (assuming perfect ideality \([22]\)). The pump transmission is given by

\[ T_p = \left| 1 + i \kappa_p \frac{a_p}{\kappa_p} \right|^2. \] (22)

This quantity can be easily calculated using Eq. (20), and different cavity loading conditions can be divided into three characteristic regimes on the basis of (22). The three regimes are overcoupled \((0 < T_p < 1, \) smaller gap width than critically coupled position), critically coupled \((T_p = 1), \) and under-coupled condition \((0 < T_p < 1, \) larger gap width than critically coupled position), respectively. The coupled threshold power can now be expressed simply as

\[ P_{\text{th}}^{\text{coupled}} = (1 - T_p)|s|_{th}^2. \] (23)

The differential slope efficiency expression can be derived from Eqs. (19) and (20) as follows:
Figure 4 shows: (a) the input threshold power; (b) the slope efficiency; and (c) the fraction of coupled input pump power, all as a function of cavity loading (gap width) and intrinsic $Q$ factor, calculated using (21), (24), and (22) using the same parameters as in Fig. 3. Figure 4(a) shows that at a given $Q$ factor, the minimum threshold power is a sensitive function of gap width as also shown in Fig. 3. The threshold increases sharply in the overcoupled regime (small gap width) due to the sharp increase in cavity loss, and in the undercoupled regime due to pump-coupling inefficiency from the fiber to the resonator. For an optimum gap width, the threshold increases for decreasing intrinsic $Q$ factor.

The slope efficiency [Fig. 4(b)] reaches $\sim 100\%$ for very high $Q$ factor ($10^{10}$) at an optimum gap width of $\sim 0.5 \, \mu m$. It drops rapidly at over- and undercoupled conditions. This optimum gap width for the lowest threshold power and highest slope efficiency in Figs. 4(a) and 4(b) corresponds to a slightly undercoupled condition, as can be seen in Fig. 4(c). This figure also shows that the critical coupling point (as inferred by the gap width) is roughly constant for high intrinsic $Q$ factors and moves towards the microcavity as intrinsic $Q$ factor decreases (or $Q < 10^8$). This change of critical coupling point is caused by the variation of intrinsic erbium-related $Q$ factor $Q_{e}^{p}$, defined in (15). As the intrinsic $Q$ factor decreases below $10^{8}$, the intrinsic erbium-related $Q$ factor becomes independent of erbium parameters and related only to the intrinsic $Q$ factor, leading to a decrease in critical coupling gap width.

Asymmetries of threshold power, slope efficiency, and the fraction of coupled power with respect to gap width of minimum/maximum value are observed in Fig. 4. These asymmetries result from the lasing condition imposed on minimum gap width for lasing (maximum achievable gain should be larger than the total round-trip loss), which will be discussed in the next section.

E. Lasing condition revisited

Equation (9) can be used to establish a lower bound for the erbium ion concentration required to obtain lasing in a cavity with a given passive $Q$ factor. Taking into account that the maximum upper-state population cannot exceed the total erbium ion concentration ($N_{2} < N_{T}$) in (9)

$$g_{s}^{*} > \alpha_{s}^{\text{passive}}.$$  \hfill (25)

Equation (25) sets the limit on the minimum erbium concentration at fixed passive intrinsic quality factor required to overcome loss. Rewriting (25) using the constituent relation given by (8) and (10), the minimum erbium ion concentration is bounded by

$$\frac{N_{2}}{N_{T}} < \frac{\alpha_{p}}{\alpha_{p} + g_{p}}.$$  \hfill (28)

which is the limit of population inversion when the pump energy is high in (18) (i.e., near the emission band).

Figure 5 is a plot of the gap width above which lasing is possible at a given erbium concentration. Data are shown for different passive cavity intrinsic quality factor $Q_{c}^{p}$ in the range from $2 \times 10^{7}$ to $1 \times 10^{8}$.

As an illustrative example, by assuming the parameters discussed above and a passive $Q$ factor of $5 \times 10^{7}$, the erbium concentration should be greater than $7.3 \times 10^{17} \, cm^{-3}$ (0.001 at. %). Under 980-nm pumping nearly complete inversion is feasible and the limits contemplated in Eq. (26) may be achieved. However, when pumping occurs in the erbium absorption band at or slightly below 1.5 $\mu m$, pump-stimulated emission must be taken into account. A tighter bound on $N_{T}$ is then obtained by following two lines of reasoning.

First, from (19), the following inequality is required for positive differential slope efficiency (i.e., lasing operation):

$$\frac{(\alpha_{s} + g_{s}^{*})}{(\alpha_{s} + g_{p})} - \alpha_{s} > \alpha_{s}^{\text{passive}}.$$  \hfill (27)

For the special case of 980-nm band pumping (at which wavelength the emission cross section is nearly zero), Eq. (27) becomes identical to (25). However, in general (27) is a stricter condition. Inequality (27) can also be obtained by setting a limit on the maximum achievable population inversion in Eq. (9)
creases and eventually becomes zero (at which point the tapered fiber and microcavity contact each other). This characteristic concentration ($3.4 \times 10^{20}$ cm$^{-3}$, 0.5 at. % in this case) above which lasing can be observed with zero gap width is independent of intrinsic $Q$ factors and determined mainly by erbium parameters and microcavity size parameters.

**F. Dependence on erbium concentration**

For the purpose of optimizing the coupled threshold power and differential slope efficiency as a function of erbium ion concentration, Eqs. (21) and (24) are evaluated. In this analysis, cooperative up-conversion or ion pairing are not taken into account. Instead, to take into account concentration quenching and subsequent lifetime shortening of the erbium ions, the concentration-dependent erbium upper-state lifetime $\tau_{\text{Er},N_T}$ is assumed to follow the reduction formula (with quenching concentration $N_Q$ of $5 \times 10^{20}$ cm$^{-3}$ assumed) given in Ref. [32]

$$\tau_{\text{Er},N_T} = \frac{\tau_{\text{Er}}}{1 + (N_d/N_Q)^2}. \quad (29)$$

Figure 6(a) shows the minimum achievable threshold power plotted as a function of erbium concentration and intrinsic $Q$ factor. For a given $Q$ factor, there is an optimal erbium ion concentration, which gives the lowest input threshold power. The higher the intrinsic $Q$ factor, the lower the optimum erbium concentration. On either side of the optimum concentration, the threshold power increases sharply with concentration. The increase on the low-concentration side is caused by condition (27); the increase on the high-concentration side is caused by concentration quenching, increase in pump transmission [reduced pumping efficiency; see Fig. 4(c)], and higher erbium concentration itself. Figure 6(b) shows the maximum achievable differential slope efficiency as a function of total erbium ion concentration and intrinsic $Q$ factor. At a given $Q$ factor, the slope efficiency initially becomes larger as the erbium concentration increases, and then becomes smaller beyond a characteristic erbium concentration of $9 \times 10^{20}$ cm$^{-3}$ (1.4 at. %). For a given $Q$ factor $>10^5$, $\sim 100\%$ efficiency can always be achieved, provided the $Q$ factor and junction ideality are sufficiently high. Note that while these calculations do take into account the concentration-quenching related reduction in lifetime at high erbium ion concentrations, cooperative up-conversion is not taken into account. This effect reduces the effective erbium ion population that can be achieved at a given pump power. Inclusion of this effect will cause a reduction in the achievable slope efficiency at given erbium ion concentration and $Q$ factor.

**G. Mode volume, external coupling efficiency, and size of toroidal microcavity**

The expressions derived in this section can be applied to any erbium-doped laser analysis. In this paper, the laser cavity corresponds to a toroidal microcavity, and the coupling, both to and from the toroidal microcavity, is achieved by evanescent-field coupling using a tapered optical fiber [22,33,34]. Because analytic expressions are not available for the coupling dependence on gap width and mode volume, the present analysis uses values in comparably sized microspheres for which there are readily available analytical expressions [20,30]. This will cause slight differences in the results. For example, the mode volume in a toroidal microcavity is smaller than that in a spherical cavity of the same diameter [35]. It is an approximately quadratic function of the major diameter of toroidal microcavity. As the lasing threshold is linear in mode volume [see Eq. (21)], the lowest lasing threshold power is expected for small toroids. For very small toroids, however, radiative (whispering-gallery) loss becomes dominant, and thus the smallest diameter of a toroidal microlaser that can be practically achieved is determined by a balance between gain (erbium concentration) and loss, given a certain available pump power. From a calculation assuming an erbium concentration of $2.5 \times 10^{20}$ cm$^{-3}$ (for which the effects of concentration quenching and cooperative up-conversion are relatively small) and the known size-dependent whispering-gallery loss [30], it follows that the smallest toroidal microcavity diameter is typically $\sim 15 \mu$m.
III. FABRICATIONS

In this section we present fabrication procedures and structural data on toroidal microcavities that were doped with erbium by ion implantation. The starting platform for the fabrication of these microcavities is an intrinsic Si(100) wafer with a 2-µm-thick SiO2 film grown by thermal oxidation. The fabrication consists of five steps involving erbium ion implantation, high-temperature thermal annealing, photolithography, wet/dry etching, and CO2 laser reflow/annealing.

Erbium ion implantation is performed with 2-MeV erbium ions at fluences of 1.2 x 10^15, 4.2 x 10^15, and 1.2 x 10^16 cm^{-2} using a Van de Graaff accelerator. The erbium doping depth profile was measured by Rutherford back-scattering spectrometry and is well fitted by a Gaussian with a peak depth of 562 nm and a standard deviation of σ = 75 nm. The peak erbium ion concentrations for the three implant fluences were 0.10, 0.35, and 1.0 at. %, respectively.

To optically activate the erbium ions, the implanted wafer was annealed at 800 °C for 1 h in vacuum. Our data demonstrate that the 800 °C thermal anneal is sufficient to restore the silica network's structure, to incorporate erbium ions onto optically active sites, and to reduce the defect density to a low level so that high intrinsic Q factor is achieved (see further on). The photoluminescence lifetime at 1.53 µm for the sample implanted with 4.2 x 10^15 cm^{-2}, measured after annealing, is 10 ms. This value is lower than the radiative lifetime of erbium ions in thermally grown SiO2 (17 ms), which is ascribed to concentration quenching due to resonant energy transfer among erbium ions followed by quenching at OH− impurity sites.

Processing of the implanted wafers into toroidal microcavities proceeds as described elsewhere [18]. Figure 1 shows the graphical rendering of the toroidal microcavity with tapered optical fiber, and the inset in Fig. 7 shows a scanning electron microscope (SEM) image of a toroidal microcavity (major diameter of 23 µm) used in the experiment. Briefly, after the high-temperature annealing process, circular photoresist pads (40–100-µm diameter) are created on the erbium-implanted silica film by standard photolithography. Next, the sample is immersed in buffered HF at room temperature to remove unpatterned silica. In this way, arrays of circular silica disks are fabricated on the silicon chip. Isolation of the silica disks from the silicon substrate is needed to prevent unwanted light leakage into the silicon substrate. This is achieved by isotropic, silicon dry etching using XeF2, leaving silica disks supported by a silicon pillar structure.

Finally, to create ultrahigh-Q toroidal microcavities, the silica disks are selectively reflowed by irradiation using a CO2 laser (λ=10.6 µm). As has been demonstrated before [18], this leads to melting and subsequent collapsing of the silica disk to a toroidal shape, with limiting dimensions defined by the width of the supporting silicon pillar. Toroidal microcavities made in this way exhibit record high-Q factors for planar devices in excess of 10^8 and therefore make excellent base resonators for the study of erbium-doped laser structures [16,17].

To study the effect of laser annealing on the optical properties of the erbium ions, we measured luminescence decay traces at 1.535 µm for both disk and toroidal microcavities (i.e., both pre- and postreflow process), and they exhibit similar lifetimes of ~10 ms [17]. In the following analysis we have assumed that as a result of the laser annealing, the erbium ions are homogeneously distributed throughout the toroidal volume, at average concentrations of 6 x 10^18, 2 x 10^19, and 6 x 10^19 cm^{-3}, for the three implant fluences, respectively (corresponding to concentrations of 0.009, 0.03, and 0.09 at. %, respectively).

IV. EXPERIMENTS

A typical high-resolution transmission spectrum measured through the fiber coupled to the erbium-implanted toroidal microcavity (4.2 x 10^15 erbium/cm^2) is shown in Fig. 7. The spectrum was measured by scanning a narrow-linewidth (<300 kHz), tunable (1400–1500 nm), external-cavity laser across a toroidal microcavity whispering-gallery resonance and monitoring the transmitted power using a photodetector (125 MHz). The frequency scanning speed (<100 Hz) of the laser was carefully chosen to satisfy both the quasi-steady-state condition imposed by the erbium ion decay rate and the maximum scanning speed condition required for the precise measurement of high-Q factors. To couple light efficiently in and out of the erbium-implanted toroidal microcavity, a tapered optical fiber (diameter of 1–2 µm) was aligned in the equatorial plane of the toroidal microcavity. Taper fabrication and coupling characteristics are described elsewhere [22,33,34]. The microcavity was attached to a piezoelectric stage (minimum step size of 20 nm) that allowed for its precise positioning with respect to the tapered optical fiber. In the spectrum of Fig. 7, the laser center wavelength was set to 1411 nm and the resulting loaded Q factor was 3.9 x 10^7. To prevent thermal broadening and spectral distortion [36] associated with the high Q, the input power was always kept below 10 µW during measurement of the resonance. Because the measurement was performed in the undercoupled regime (~79 % transmission) and well below the lasing threshold, i.e., with the major fraction of the erbium ions in the ground state 4I_{15/2}, the observed linewidth provides a good estimate of the unloaded Q factor. The erbium absorp-
tion related contribution to $Q$ factor can be estimated using (17) and yields $Q_{wp} = 3.25 \times 10^7$ (assuming $N_T = 2 \times 10^{19}$ cm$^{-3}$, $\sigma_p = 1 \times 10^{-22}$ cm$^2$, $n_p = 1.46$, and $\Gamma_p = 1$). It is therefore believed that the measured $Q$ factor is predominantly caused by the absorption of erbium ions.

To confirm the inverse relation between weakly pumped intrinsic $Q$ factor and erbium ion concentration given by (16) and (17), weakly pumped $Q$ factors at three different concentrations were measured as shown in Fig. 8. A complication that arises when comparing $Q$ factors for different resonators is that all resonators have different resonance wavelengths, determined by their slightly different geometry. In order to compare $Q$ factors, they have been normalized to values at a wavelength of 1450 nm, using the known erbium absorption cross-section spectrum [25] as a(n) (inverse) normalization factor. The line in Fig. 8 provides the expected inverse relation between $Q$ factor and erbium concentration and fits the data well. This further confirms that the measured $Q$ factors are dominated by erbium-related absorption. The inset of Fig. 8 shows a side-view color optical micrograph of a toroidal microcavity with implantation fluence of $1.2 \times 10^{16}$ cm$^{-2}$.

Figure 9 shows the measured output lasing power versus fiber-coupled, input pump power for a microlaser implanted with $4.2 \times 10^{15}$ erbium/cm$^2$. To pump the microlaser, the same laser used to measure the absorption spectrum in Fig. 7 was tuned to a toroidal microcavity resonance within the absorption band (1460–1480 nm) of the erbium ions. Input pump power and lasing output power were carefully measured in order to account for modest amounts of tapered fiber loss. Two sets of measurements corresponding to two differ-
Different toroidal microcavities are presented in Fig. 9. These devices illustrate two distinctive operating conditions (low threshold and high output power). The order of magnitude difference in threshold powers of these devices is attributed to the difference in the size of microcavities as well as in the $Q$ factor. In Fig. 9(a), a threshold power of 4.5 $\mu$W is observed for a 25-$\mu$m-diameter toroidal microcavity (lasing wavelength of 1591.3 nm and pump wavelength of 1472.1 nm). The pump transmission was about $\sim$50% in this measurement, so the actual coupled threshold pump power is estimated to be approximately 2–3 $\mu$W. This is, to the best of the authors’ knowledge, the lowest threshold power reported for any erbium-doped laser. In this case, the $Q$ factor was so high that even at very low input pump power ($<100$ $\mu$W), thermally induced resonance broadening [36] and associated increase in pump power transmission (due to a finite frequency scanning range in experiment) resulted in a saturated output lasing power which prohibited further output power increase.

Figure 9(b) shows a measurement with relatively high output lasing power (unidirectional slope efficiency=2.6%) measured using a 50-$\mu$m-diameter toroidal microcavity. The measured output power was 29 $\mu$W and the input threshold power was 116 $\mu$W. From a series of experiments on the same microcavity and for different cavity loading conditions, the highest (unidirectional) output power measured was 39.4 $\mu$W and the highest unidirectional slope efficiency was about 5.5 %. The dependence of threshold, slope efficiency, and lasing wavelength on cavity loading (i.e., the gap width) by the tapered fiber was also investigated. Figure 10 contains a series of lasing wavelength scans under varying loading conditions and illustrates the effect of increased cavity loading on oscillation wavelength. This measurement was performed using the same microcavity laser of Fig. 9(b). At large gap width (corresponding to the undercoupled or weakly loaded regime), lasing at relatively longer wavelengths (1588.6 nm) is observed. As the gap width decreases (cavity loading increases), the lasing wavelength shifts in discrete steps to shorter wavelengths, enabling us to broadly tune the lasing wavelength by the adjustment of cavity loading. This effect is believed to be due to the cavity-loading-induced change in the inversion of erbium ions [Eq. (9)]. In particular, as the loading increases (gap width decreases), and thus loaded $Q$ factor decreases, the oscillation wavelength must adjust to shorter wavelengths where the available erbium gain is larger (assuming that the $Q$ factors are the same for all the whispering-gallery modes in the erbium gain bandwidth). This kind of discrete wavelength tuning was possible only in microcavities of moderate sizes. For larger cavity diameter (>70–80 $\mu$m), and hence smaller free spectral range (FSR), multimode lasing was dominant. For smaller diameters below ~40 $\mu$m, the FSR is so large that within the erbium gain spectrum only one lasing wavelength satisfies the lasing condition, at an erbium upper-state population determined by the gap width.

The input threshold power and unidirectional slope efficiency were investigated as a function of cavity loading for two lasing wavelengths, 1557.1 and 1588.6 nm. Figure 11 shows the input threshold power as a function of gap width between the tapered fiber and the microcavity. In the undercoupled regime, lasing starts at a relatively long wavelength (1588.6 nm) first with high threshold due to the small pump coupling. As the gap width decreases, the threshold decreases at first, but ultimately increases abruptly near the gap-distance limit ($\sim$1.1 $\mu$m) defined by the condition (27). Eventually lasing at 1588.6 nm can no longer be supported because the increased cavity loading (and hence reduced $Q$ factor) cannot be compensated by the relatively small gain at this wavelength (small emission cross section). As the coupling distance is further reduced, the next lasing line at 1557.1 nm appears and similarly, the threshold power is observed to increase near the gap-distance limit ($\sim$0.4 $\mu$m for this wavelength).
The inset in Fig. 11 shows the differential slope efficiency as a function of gap width for the two lasing wavelengths. Starting from the weakly loaded regime, the differential slope efficiency increases as the cavity loading increases and reaches a maximum, in qualitative agreement with the calculations in Fig. 4(b). The same is observed for the second lasing line that appears with increased loading. For each lasing wavelength, the lowest threshold and highest slope efficiency are achieved at the same optimum gap width, in agreement with Fig. 4.

Figure 12 shows the threshold power measured for toroidal microcavities having three different erbium concentrations. The top axis shows the erbium concentration assuming erbium ions are homogeneously distributed in the toroidal microcavity. In each case, multiple measurements were performed on a series of toroidal microcavities with the same erbium ion fluence, and the lowest measured threshold was plotted in Fig. 12. The data show only a weak dependence on erbium concentration. This, and the absolute values of the measured thresholds, is in qualitative agreement with Fig. 6, which shows a weak dependence on concentration in the concentration range $7 \times 10^{18} - 3 \times 10^{19}$ cm$^{-3}$ for an intrinsic $Q$ of $10^5$, typical for our experiments.

V. CONCLUSIONS

Lasing in erbium-implanted high-$Q$ silica toroidal microcavities coupled to a tapered fiber has been investigated analytically and experimentally. Threshold powers as low as 4.5 $\mu$W and output power of 39.4 $\mu$W are experimentally achieved using differently sized resonators and coupling conditions. Controlling the lasing wavelength was possible by changing the tapered-fiber-microcavity gap width. Formulas predicting various performance metrics such as threshold power, differential slope efficiency, optimum concentration of erbium ions, as well as the influence of resonator size and the $Q$ factors were derived and the calculations are in good agreement with the experimentally obtained results.

ACKNOWLEDGMENTS

This work was supported by the Defense Advanced Research Project Agency, the National Science Foundation, and the Caltech Lee Center. The Dutch part of this work was part of the research program of the Foundation for Fundamental Research on Matter (FOM) and was financially supported by the Dutch Organization for Fundamental Research on Matter.