Nanophotonics: Shrinking light-based technology

A. Femius Koenderink,† Andrea Alù,‡,§ Albert Polman*†

The study of light at the nanoscale has become a vibrant field of research, as researchers now master the flow of light at length scales far below the optical wavelength, largely surpassing the classical limits imposed by diffraction. Using metallic and dielectric nanostructures precisely sculpted into two-dimensional (2D) and 3D nanarchitectures, light can be scattered, refracted, confined, filtered, and processed in fascinating new ways that are impossible to achieve with natural materials and in conventional geometries. This control over light at the nanoscale has not only unveiled a plethora of new phenomena but has also led to a variety of relevant applications, including new venues for integrated circuitry, optical computing, solar, and medical technologies, setting high expectations for many novel discoveries in the years to come.

Optics and the science of light is a lively field of research that continues to surprise decade after decade, with fundamental breakthroughs and disruptive applications. Communications technology has been revolutionized by the invention of the laser and the optical fiber, incandescent light bulbs are being replaced by efficient solid-state lighting, and solar energy technologies are on their way to price parity with fossil fuel-based power generation. A large number of these developments have resulted from increased control over the flow of light at length scales smaller than the wavelength. Squeezing light to nanoscale dimensions also opens the prospect of dense optical integrated circuits, which may overcome fundamental challenges related to bandwidth and energy dissipation in today’s electronic integrated circuit technology. More broadly, the field of nanophotonics aims at overcoming Abbe’s diffraction limit, developing technology able to manipulate light on a deep-subwavelength scale. As photons are shrunk to the nanometer scale, ultimately approaching the scale of the wave function of electrons, fundamental new science is expected, and important technological advances appear. In this article, we review recent highlights in the science and applications of nanophotonics, focusing on the ultraviolet (UV)/visible/near-infrared spectral range, and provide an outlook for the bright future of this research field.

Photonic crystals

The initial concept for on-chip miniaturization of light dates back to the late 1990s, when photonic crystals—periodic structures fabricated from high-refractive-index materials such as Si or GaAs—were proposed and realized (Fig. 1A).

As the periodicity in these structures approaches the wavelength of light, a photonic bandgap can appear, analogous to the energy bandgap in a semiconductor. The propagation of light with a frequency in the band gap is then forbidden, except in localized regions created by a well-designed break in periodicity, such as line defects that can guide light, or point defects that confine light. Band structure engineering provides exquisite control over light dispersion—that is, over the relation between its frequency ω and its effective propagation constant k = 2π/λ;—and thereby also over how fast signals of different wavelengths propagate, as given by the group velocity dω/dk. Slowing down light in waveguides, and confining it in optical nanocavities, can be used to create optical memories and enhance light switching and manipulation schemes based on ultrafast modulation of the local refractive index. Impressively, data transmission with just one femto-Joule of energy expenditure per bit was recently demonstrated by using this photonic crystal platform, and as many as 100 individually addressable and switchable optical cavities have been multiplexed to reach a low-power optical memory with nanosecond storage time (1).

Plasmonics

Confining light waves is much more difficult than confining electron waves because there is no optical equivalent to the Coulomb force, based on which deep electron traps can be created. Instead, the only parameter to shape light is the material dielectric constant (ε = n², where n is the refractive index), which is limited to the range n = 1.3 to 4.0 for nearly all dielectrics in the visible

![Fig. 1. Nanophotonic architectures. (A) 2D photonic crystal waveguide coupled to resonant cavities serves as a wavelength division multiplexer. (B) Metal-insulator-metal surface plasmon polariton waveguide strongly confines light and shrinks the wavelength. (C) Plasmonic dimer nanocantenna coupled to an optical emitter creates directional emission of light. (D) Metasurface composed of chiral antennas offers selectivity to circularly polarized light. (E) Metasurface composed of graded plasmonic or dielectric antenna geometries enables wavelength-dependent control over the reflection and refraction of the optical wave front.](image-url)
Angular distribution of radiation. Yet, tools borrowed from radio-frequency antenna design have allowed far-reaching control over the directivity of light radiation and scattering from proper arrangements of nanoantennas, and conversely over how far-field radiation drives near-field focusing in properly designed nanochannels. Following these principles, antennas coupled to a single-quantum emitter can result in highly directional beaming of spontaneous emission (6), and phased-array antennas coupled to efficient single-photon sources can serve as light-steering elements. Over 1000-fold brightness enhancement per fluorophore has been recorded in single-molecule experiments aimed at improving fluorescence microscopy in biological systems (7). Besides engineering emission directivity, antennas open a completely new regime for light-matter interaction strengths, especially in the case of dimer antennas with narrow gaps, through which plasmon modes can efficiently interact (Fig. 1C). A quantitative measure of the light-matter interaction enhancement around a nanoantenna is the Purcell factor: the spontaneous emission rate increase due to enhanced photon mode density in the antenna's optical near field. Recently, a Purcell factor as high as 1000 was reported in metallic nanoantennas with gaps in the order of just a few nanometers (8). The capability of plasmon antennas to transduce between light and molecules has large relevance for microscopy, such as in the case of observation of single molecules, and infrared vibrational spectroscopy by use of field enhancement at sharp metal tips (9).

Shrinking vector fields to boost magnetism and chirality
Antennas not only localize the photon energy density but also provide a way to control the vector nature of light at the nanoscale. The relevance of magnetic effects in optics is conventionally neglected for small objects because material magnetization rapidly vanishes as the frequency grows. This also implies that chiral effects such as circular dichroism and optical rotary power are inherently weak in ordinary matter because they require that distinct optical responses—such as an electric dipole plus a magnetic dipole response, or an electric dipole plus a quadrupole response—occur simultaneously in the same object with a controlled phase lag. Such magnetic and magnetoelectric effects can be boosted to become as strong as conventional electric responses by setting up scattering resonances in loop-shaped and helical antennas that carry circulating optical currents, or by pairing achiral resonances in strongly asymmetric geometries (17, 18). These nanostructures may allow enhancement in the local chirality of the optical field so that enantiomers—molecular stereoisomers that are mirror images of each other—may be optically screened at the single-molecule level by placing them near a chiral antenna (19). Furthermore, helicity-dependent near-field enhancements may be used to control spin-polarized optical transitions.

Metamaterials and metasurfaces
The unprecedented opportunity to localize light at deep-subwavelength scales has not only been applied to isolated optical nanoresonators but has also led to the synthesis of optical metamaterials, artificial materials with an unusual optical response, formed by ordered or disordered collections of resonant nanoscale plasmonic scattering elements. Many unusual bulk optical responses have been theoretically predicated and experimentally verified on the basis of complex three-dimensional (3D) metallo-dielectric architectures by using nanoantennas as their basic elements. Remarkable nanofabrication advances

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have allowed the realization of nanostructured materials with unusual optical responses such as, for example, a metamaterial slab composed of 3D chiral helix antennas (Fig. 1D) (20), offering large selectivity to circular polarization with a broadband response in an optical frequency band in which optical components filtering circular polarization do not exist. The fact that constituent antennas can carry both a strong electric and a strong magnetic response gives the opportunity to reach optical properties far outside the scope of naturally available refractive indices, permittivities, and permeabilities. Such materials can, for example, refract light in unexpected directions, as demonstrated in various geometries, most recently in the UV spectral range (21). Negative refraction has been a precursor to the broad paradigm of transformation optics (22), which is most popular for enabling invisibility cloaks that can make objects undetectable in a certain frequency band by wrapping them in a metamaterial shell with suitably graded electric and magnetic response. Strong frequency dispersion, as well as absorption that takes place in the constituent antennas, are fundamental constraints of passive metamaterials that have so far limited their practical applicability.

The need to overcome loss, combined with the continuous drive toward integration and large-area fabrication, has inspired a recent shift from 3D metamaterials toward 2D optical metasurfaces (Fig. 1E). Metasurfaces are planarized, ultrathin, patterned artificial surfaces that are designed to mimic the functionalities of conventional optics and metamaterials in two dimensions, avoiding absorption losses by light propagation in the third dimension. Strongly localized optical resonances induced by plasmonic nanoantennas enable abrupt phase and amplitude discontinuities, which can be tailored at will across the surface to provide a controllable transverse gradient, inducing anomalous refraction, reflection, and control over subwavelength structure of the impinging wavefront (23, 24). This enables the realization of ultrathin lenses, beam steering devices, and generation of orbital angular momentum of light, to name just a few examples. A promising trend in metasurface design is the use of arrays of dielectric nanocircuit elements that show geometric Mie-type resonances, avoiding absorption losses (25, 26).

Recent work has shown that flat and ultrathin metamaterials may be used to engineer to a large extent both the spatial and spectral response of the impinging wavefront. In this sense, metasurfaces and metamaterials can operate as all-optical circuitry, processing the impinging signals at the nanoscale with nanocircuit elements that may form thin metasurfaces acting as complex operators (27). These concepts may lead to all-analog filtering, signal processing, and even computing functionalities performed as light interacts with these devices (28).

Probing nanoscale optical fields

Measuring light confinement is inherently difficult because the diffraction limit of conventional microscopy places a lower limit to imaging resolution. Yet, advances in nanophotonics research require new techniques to excite materials and probe them at the nanoscale. Although super-resolution microscopy tools such as photoactivated localization microscopy (PALM), stochastic optical reconstruction microscopy (STORM), and stimulated emission depletion (STED) have made it possible to circumvent Abbe’s limit, they work best for specimens that are almost transparent and that can be functionalized with proper fluorescent markers. Nanophotonics research instead often requires a point detector, or a point source of vector fields that can be brought into the optical near field. Near-field scanning microscopy uses a sharp raster scanning probe to approach a nanostructure of interest to within 10 nm from its surface. The probe either acts as a scatterer that converts near fields into far-field light (4, 5) or as a fiber-coupled source or detector. This technique has become so advanced in recent years that electric and magnetic field components of light can be separately imaged at a length scale far below the wavelength, and at femtosecond time scales (Fig. 2A) (29). Scanning emitter lifetime imaging microscopy can probe the optical density of states in two dimensions by use of a single fluorescent source attached to the end of a sharp fiber probe (30). Electron beam–induced excitation in a scanning transmission electron microscope is another highly controllable way to study optical modes and resonances in polarizable metallic and dielectric nanostructures with deep-subwavelength spatial resolution (Fig. 2B). Cathodoluminescence spectroscopy enables spatial mapping of the radiative density of states in two dimensions by using the light emitted when a beam of fast electrons impinges on a sample (31). Conversely, electron energy loss spectroscopy (EELS) maps the kinetic energy lost by electrons during their interaction with photonic structures. EELS and CL have recently been demonstrated in tomography mode, imaging the localized surface plasmon modal field distribution of Au nanoparticles in three dimensions (32, 33). Electron beam–induced optical images routinely provide a spatial resolution of ~5 nm, which is on par with the best optical super-resolution microscopy.

Practical applications of nanophotonics

Nanophotonics has already delivered many of the original promises dating back to when this research field started to develop around 10 years ago. One of the earliest nanophotonics discoveries to transcend the laboratory were chemically synthesized silica-core/Au-shell nanoparticles for applications in medical diagnostics and therapy (34). When introduced into the blood stream, these “nanoshells” are preferentially trapped in malicious tissues. When irradiated with a laser tuned to their plasmon resonance, the particles are locally heated, destroying the cells. This concept is presently being tested in clinical trials on humans for cancer treatment.

Molecular sensing and spectroscopy have motivated the drive to controllably shrink light to the size of a single molecule. It was found that optical hotspots in rough metal surfaces arising from localized surface plasmons result in a strong enhancement of the molecular Raman scattering signal, forming the basis for surface-enhanced Raman scattering (SERS), a well-established spectroscopy tool in chemistry. In a similar area, new sensors have been proposed based on the plasmon resonance wavelength shift caused by different molecular species placed in the plasmonic near field. Antibody-specific chemistry may also be used to bind or cluster plasmon particles by using the fact that gold is easily functionalized through thiol chemistry; commercial pregnancy tests today use this concept. An intriguing application of plasmonic light focusing is in DNA sequencing, in which a DNA molecule is transported through a small hole in a metal film, with the aim to read out the base-pair sequence through the subsequent detection of fluorescent markers that are selectively bound to different base pairs (35).

Lasers, solid-state lighting, and photovoltaics are also important fields in which nanophotonic
structures have enabled new designs for improved functionality. Photonic crystal lasers have now become so advanced that they can deliver power in the watt range (Fig. 3A) (36). The emission of light-emitting diodes (LEDs) is strongly enhanced if a suitably designed periodic array of Ag nanoparticles is embedded in the light-emitting phosphor (Fig. 3B). These nanostructures help to efficiently couple and scatter light from the UV pump LED into the phosphor material and, at the same time, aid the directional outcoupling of the phosphor emission, which enhances LED efficiency and brightness (37). Conversely, periodic and aperiodic metasurfaces composed of resonant plasmonic or dielectric nanoparticles can lead to improved light coupling and trapping into solar cells, supporting increased photovoltaic energy conversion efficiency, as well as thinner cell designs that can be made at lower cost (Fig. 3C) (25). Recently, efficient solar cells were realized by using InP nanowires (38). Here, the small radius of the wire enables efficient collection of electrical carriers. At the same time, optical resonances in the nanowire can lead to light concentration, increasing the photovoltage (39).

Metal nanowire networks have been developed as transparent electrically conducting coatings. Even though these nanowires suffer from ohmic losses owing to plasmon excitation, they show a beneficial tradeoff between optical transmission and electrical conduction and can be made at relatively low cost (Fig. 3D) (40). These nanowire networks are already finding applications in solar cells, computer display, and tablet technology, replacing the commonly used, expensive, and brittle indium-tin-oxide as a transparent top-contact.

The near-field focusing provided by plasmonic nanostructures has also found applications in heat-assisted magnetic recording (HAMR) for data storage, a technique in which the magnetic phase change in a recording medium is facilitated by a transient temperature increase (Fig. 3E). This increase is induced by an optical hotspot resulting from nanofocusing of plasmons onto a magnetic film (41). Plasmonic hole arrays have been demonstrated as color multiplexers in charge-coupled display (CCD) imaging systems (Fig. 3F) (42). Here, light within specific wavelength bands is guided to the matching light collection pixels on a CCD array and then converted to electrical signals. This represents an important advantage over the use of conventional color filters, in which a substantial portion of the impinging light is lost by absorption.

**Future developments and perspectives**

Nanophotonics provides a diverse set of tools to build on: Photonic crystals offer ultimate dispersion control and low-loss storage, whereas plasmonics is the platform of choice to control light on ultrafast time scales and ultrasmall length scales, matching optical interactions with single molecules. Metamaterials and metasurfaces provide the ultimate control over all properties of light. With the great control over light at the nanoscale that has now been achieved, the nanophotonics community is in an ideal position to bring the field another step further, by coupling light with other degrees of freedom at the nanoscale. Hybrid nanophotonics revolves around the simultaneous control of tightly confined light and phonons, electrons, spins, and/or excitons interacting with light. For example, when optical fields and acoustic vibrations are colocated at the nanoscale, light can be used to control mechanical motion and vice versa, with optomechanical coupling strengths that are not achievable with other geometries (Fig. 4A). These interactions have, for example, been used for laser cooling of a nanomechanical
resonator to its quantum ground state (43). It has even been suggested that plasmon-enhanced SERS can be described by the dynamic backaction of the plasmon on a molecule’s vibration, paving the way to a new form of molecular quantum optomechanics (44). Future hybrid nanophotonic systems hold the promise to couple electron spins with light, enabling integrated networks for quantum nanophotonics—for example, by using single-photon emission by color centers in diamond or defect centers in materials such as SiC (45).

Nanophotonics is also at the brink of making key contributions to the development of novel energy-conversion mechanisms. The recently discovered plasmolectric effect in metal nanoparticles and hole arrays (46), in which optical illumination directly creates an electric potential by off-resonant excitation of a plasmonic structure (Fig. 4B), is being further explored to investigate how electrical power can be best generated. Another intriguing challenge is to harvest the excess energy of hot electrons excited in optically excited plasmonic nanostructures (47). This may find applications in energy harvesting and also plasmon-assisted surface catalysis for the generation of fuel (such as ethanol and hydrogen) from sunlight. More generally, plasmon-assisted photochemistry and catalysis is a research field with many opportunities to be explored.

An initial motivating factor for nanophotonics research was its direct impact on optoelectronic integration, aiming to bring together electronic and photonic length scales. The development of novel nanostructures that enable a nonreciprocal flow of light paves the way to on-chip all-optical isolation, in which light can only propagate in one direction, which is one of the missing components toward a fully integrated light-based communication system. Recent proposals and experimental demonstrations have shown that nonreciprocity can be indeed achieved by using a suitably designed photonic temporal modulation of the local permittivity in waveguides and microring resonators (Fig. 4C) (48, 49). Another interesting direction in this context lies in the exploration of the photonic equivalent of topological insulators. In such structures, unidirectional flow of light and reflectionless propagation robust to disorder may be achieved by designing periodic metamaterials such as SiC (50). As a result of these advancements, novel transistorless optical devices that support topologically nontrivial band diagrams (50).

Two further important ingredients for optoelectronic integration are access to strong nonlinearities within a very compact footprint and dynamic tunability and adaptability (51). The nonlinear optical response of materials is typically weak, and therefore large volumes are required to achieve a measurable response. The recent observation of a giant nonlinear response from plasmonic metasurfaces coupled to interband transitions in semiconductors may open exciting venues for nonlinear nanophotonics (52). Similarly, one may ask how metasurfaces can enable new schemes for dynamically tunable integrated photonics. For example, can electrically tunable metasurfaces be used to steer and multiplex light on a chip, integrating an optical fiber communication system within an electronic integrated circuit? Also, how can “metatronic” optical signal processing and computing circuits be built and reconfigured by using building blocks with discrete optical functions (Fig. 4D)? Novel 2D materials and heterostructures based on graphene, MoS2, and WSe2 will play an important role in bringing light to the nanoscale, enabling 2D electrically tunable integrated optical nanocircuits (Fig. 4E) (53, 54). It will be interesting to see to what degree of integration complexity optical and electronic functionality can ultimately be achieved on a chip by using these materials. A decisive factor will be to find the right trade-off between light confinement and optical absorption losses.

The control over light at the nanoscale that scientists have achieved in recent years leads to a continuous stream of new fundamental insights in the interaction of light with matter at a deep-subwavelength scale. There is no doubt that this intense research activity promises a bright future for nanophotonics in the years to come.

Fig. 4. Nanophotonics future challenges and research fields. (A) Hybrid nanophotonics: coupling light with other degrees of freedom, such as mechanical motion. (B) Plasmolectric effect converting sunlight to electrical power by using metallic nanostructures. (C) Nonreciprocal optical integrated circuit enables logic functions for optical computing. (D) Electrically tunable optical metasurface serves as wavelength division multiplexer in optical communication networks and may be operated as an analog computing metasurface. (E) Electrically tunable 2D graphene optoelectronic integrated circuit brings together optical and electronic length scales. Illustrations are inspired by (23, 24, 27, 28, 43, 46, 49, 54)]

REFERENCES AND NOTES
The nature of light was revealed by Maxwell a century and a half ago. His equations stand today as an exact description of light, and all electromagnetic phenomena, at the classical level. It has long been known that Maxwell’s equations are invariant under a coordinate transformation (1–3). If \( \mathbf{\varepsilon(x_1)} \mathbf{\mu(x)} \) are the permittivity and permeability tensors expressed in some coordinate frame, then a transformation to a new coordinate frame \( \mathbf{x} \) does not alter the form of Maxwell’s equations but only changes the values of permittivity and permeability to be used. Formally we have

\[
\mathbf{\varepsilon}^{\prime} \mathbf{\mu}^{\prime} = [\det(\mathbf{\Lambda})]^{-1} \mathbf{\Lambda}^{\prime} \mathbf{\varepsilon}^{\prime} \mathbf{\mu}^{\prime}
\]

where

\[
\mathbf{\Lambda}^{\prime} = \frac{\partial \mathbf{x}}{\partial \mathbf{x}^0}
\]

This simple mathematical statement has been elaborated over the past couple of decades into an intuitive working tool of electromagnetic theory (4–8). Here we give a physical interpretation of the equations and re-derive the formulae in an intuitive fashion, followed by examples of where transformation optics has been effectively applied.

Balz mathematical statements hide the real power of the transformation method, which is only revealed when we appeal to Michael Faraday’s representation of electric and magnetic fields in terms of lines of force. In this scheme, the displacement field \( \mathbf{D(x)} \) and the magnetic field \( \mathbf{B(x)} \) are represented by field lines that begin and end on charges in the case of the displacement field and are continuous in the case of the magnetic field. Their density represents the field strength. A coordinate transformation can be thought of as a distortion of space, which moves the field lines around as if they were embedded in a block of rubber. As we distort the coordinates, we carry the field lines along too. This yields an intuitive picture of how to manipulate fields: Just as Snell’s law tells how rays of light can be redirected and focused using the refractive index, transformation optics shows how field lines can be manipulated and gives an exact prescription for the values of \( \mathbf{\varepsilon(x)} \mathbf{\mu(x)} \), ensuring that the distorted field lines obey Maxwell’s equations. Replacing the rays of Snell’s law with Faraday’s fields extends our powers of visualization into regimes untouched by Snell, such as the subwavelength electric fields found in plasmonic systems and even the regime of static electric and magnetic fields (9).

Not only does this manipulation apply to \( \mathbf{D} \) and \( \mathbf{B} \), but also to any electromagnetic quantity that is conserved and therefore represented by field lines: For example the Poynting vector representing the flow of energy serves as a generalization of the ray picture. Another example is the flow of charge either as an electrical current or as the trajectory of an individual particle.

**The power of transformation**

Here we give a few examples that demonstrate the power of the technique, but first let us make an alternate, intuitive, derivation of the transformation equations.

Consider an electric displacement field \( D_y \) parallel to the \( y \) axis in a dielectric medium (as shown red in Fig. 1A). Next make a coordinate transformation in which one half of the space is uniformly compressed by a factor \( m \) (Fig. 1B). We then ask how the permittivity in the compressed medium must change. To calculate \( e_y^{\prime} \) we require that \( E_y \) be continuous. Noting that owing to the compression, the lines of force have been pushed closer together and hence \( D_y \) has been increased by a factor \( m^2 \), we find \( D_y^{\prime}/e_y^{\prime} = m \) and hence \( e_y^{\prime} = e_y^{\prime} \). The same argument for magnetic fields shows that \( u_y^{\prime} = m \). To find \( e_x^{\prime} \) consider an electric displacement field \( D_x \) parallel to the \( x \) axis (as shown in cyan

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**REVIEW**

**Transforming the optical landscape**

J. B. Pendry,¹* Yu Luo,² Rongkou Zhao³

Electromagnetism provides us with some of the most powerful tools in science, encompassing lasers, optical microscopes, magnetic resonance imaging scanners, radar, and a host of other techniques. To understand and develop the technology requires more than a set of formal equations. Scientists and engineers have to form a vivid picture that fires their imaginations and enables intuition to play a full role in the process of invention. It is to this end that transformation optics has been developed, exploiting Faraday’s picture of electric and magnetic fields as lines of force, which can be manipulated by the electromagnetic and magnetic permeability of surrounding materials. Transformation optics says what has to be done to place the lines of force where we want them to be.

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