

Some nanophotonics basics

Summerschool Plasmonics
Porquerolles, September 4-8, 2017

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Nanophotonics basics

Three building blocks of nanophotonic materials

1) Refractive index of a dielectric

What determines it?

2) Plasmonic scatterers

What is the difference between

1) bulk plasmon – 2) surface plasmon – 3) particle plasmon?

How to derive resonances and dispersion?

3) Dielectric scatterers

What is a Mie resonance & how to use it?

Refractive index of a dielectric: quick tutorial (1/3)

1) Equation of motion:

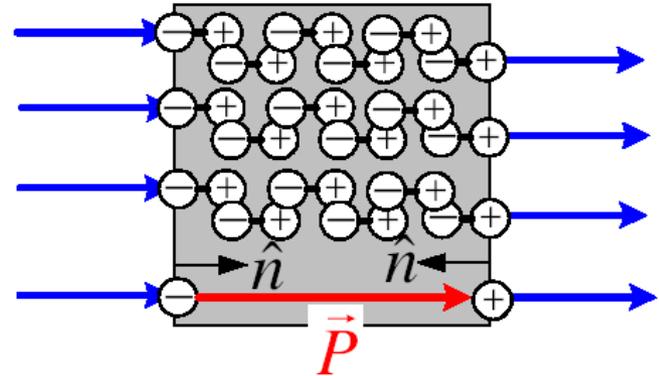
$$m \frac{d^2 \mathbf{x}}{dt^2} + m\gamma \frac{d\mathbf{x}}{dt} + k\mathbf{x} = -e\mathbf{E}$$

$$\mathbf{E} = \mathbf{E}_0 e^{-i\omega t}$$

2) Solution:

$$\mathbf{x}_0 = \frac{1}{m(\omega_0^2 - \omega^2 - i\omega\gamma)} \mathbf{E}_0$$

With: $\omega_0 = \sqrt{k/m}$



3) Polarizability

$$\mathbf{P} = -Ne\mathbf{x} = \frac{Ne^2/m}{\omega_0^2 - \omega^2 - i\omega\gamma} \mathbf{E}$$

Refractive index of a dielectric: quick tutorial (2/3)

1) Wave equation:
$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} + \mu_0 \frac{\partial \mathbf{J}}{\partial t}$$

2) Fill in:
$$\mathbf{P} = -Ne\mathbf{x} = \frac{Ne^2 / m}{\omega_0^2 - \omega^2 - i\omega\gamma} \mathbf{E}$$

3) And get
$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \left(1 + \frac{Ne^2}{m\epsilon_0} \left[\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \right] \right) \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

4) So that: with
$$\mathbf{E} = \mathbf{E}_0 e^{i(k_z z - \omega t)}$$

$$k_z^2 = \left(\frac{\omega}{c} \right)^2 \left(1 + \frac{Ne^2}{m\epsilon_0} \left[\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \right] \right)$$

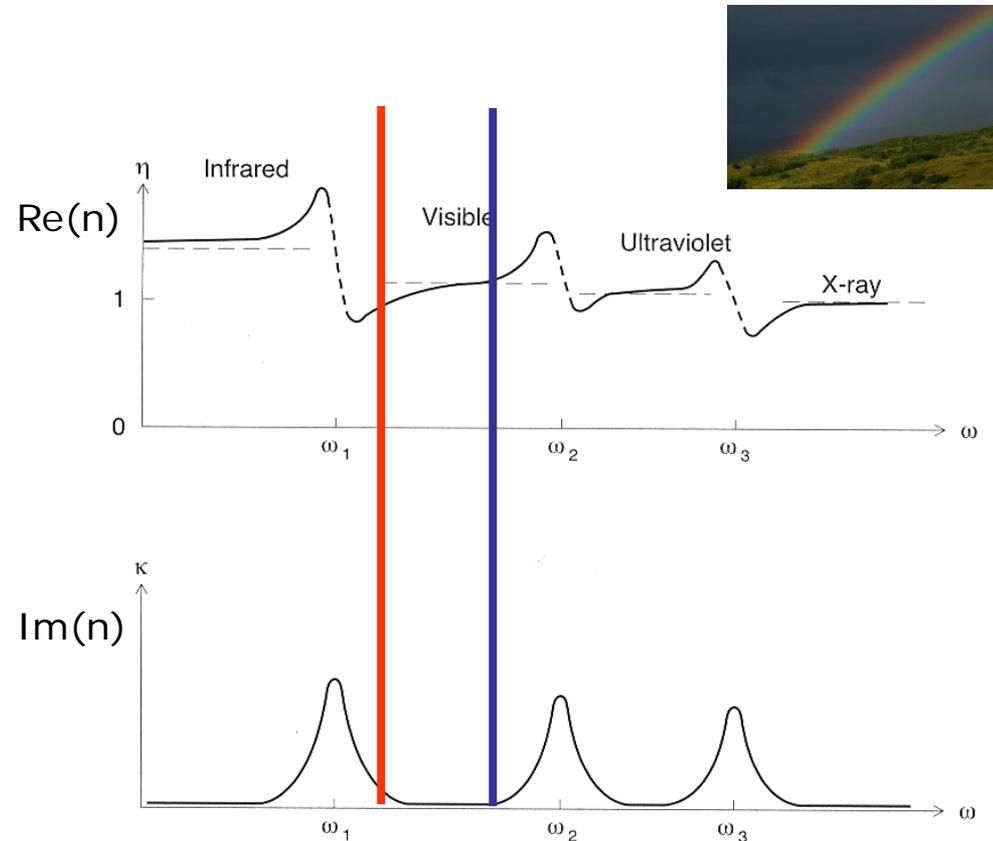
$$n = \frac{ck_z}{\omega}$$

Refractive index of a dielectric: quick tutorial (3/3)

$$n^2 = \left(1 + \frac{Ne^2}{m\epsilon_0} \left[\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \right] \right)$$

For a dielectric with multiple resonances:

$$n^2 = 1 + \frac{Ne^2}{m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}$$



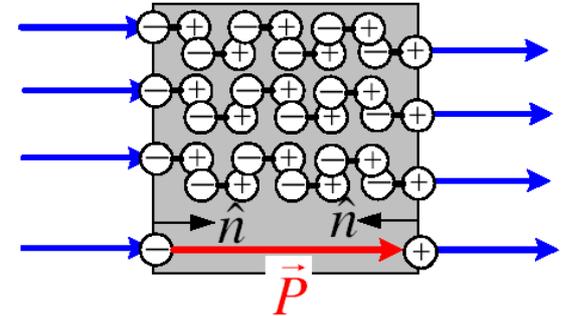
- Dielectrics are dispersive: $n(\omega)$
- Static dielectric constant is determined by high- ω resonances
- Absorption resonances coincide with index wiggles

Dielectric constant of noble metals: quick tutorial (1/2)

1) Equation of motion (Drude model):

$$m \frac{d^2 \mathbf{x}}{dt^2} + m\gamma \frac{d\mathbf{x}}{dt} + \cancel{k\mathbf{x}} = -e\mathbf{E}$$

$$\mathbf{E} = \mathbf{E}_0 e^{-i\omega t}$$



2) Solution:

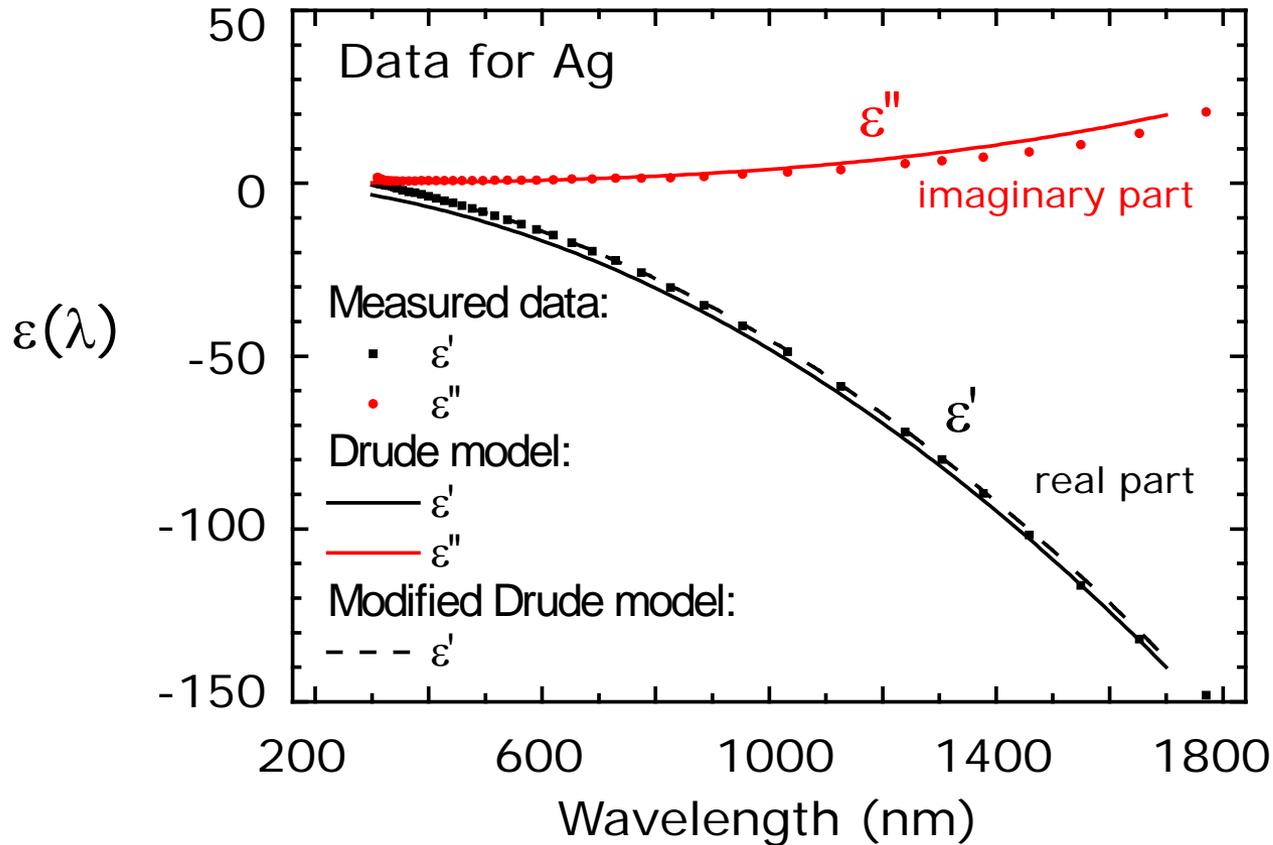
$$x = \frac{eE_0}{m(-\omega^2 + i\omega\gamma)} e^{i\omega t}$$

$$\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_0}} \quad \text{bulk plasmon resonance (EELS)}$$

3) So that:

$$\epsilon = 1 + \frac{P}{\epsilon_0 E} = 1 + \frac{Nex}{\epsilon_0 E} = 1 - \frac{Ne^2 x}{m\epsilon_0 (\omega^2 - i\omega\gamma)} = 1 - \frac{\omega_p^2}{(\omega^2 - i\omega\gamma)}$$

Dielectric constant of noble metals: quick tutorial (2/2)



Noble metals (Ag, Au, Cu, Al) have strongly negative $\text{Re}(\varepsilon(\omega))$

Relation between n and ε (1/1)

$$\varepsilon = \varepsilon' + i\varepsilon''$$

$$n = n' + in'' \quad (\text{or: index} = n + ik)$$

$$n = \sqrt{\varepsilon}$$

$$n' = \sqrt{\frac{\sqrt{(\varepsilon')^2 + (\varepsilon'')^2} + \varepsilon'}{2}}$$

$$\varepsilon' = (n')^2 - (n'')^2$$

$$\varepsilon'' = 2n'n''$$

$$n'' = \sqrt{\frac{\sqrt{(\varepsilon')^2 + (\varepsilon'')^2} - \varepsilon'}{2}}$$

Plasmon resonance of metal nanoparticles: quick tutorial (1/3)

Boundary conditions:

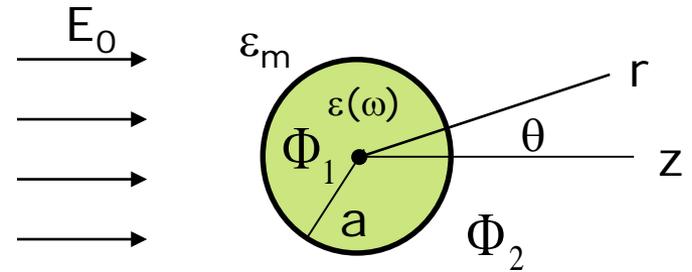
$$1) \quad \Phi_1 = \Phi_2 \quad (r = a)$$

$$2) \quad \varepsilon(\omega) \frac{\partial \Phi_1}{\partial r} = \varepsilon_m \frac{\partial \Phi_2}{\partial r} \quad (r = a), \quad \lim_{r \rightarrow \infty} \Phi_2 = -E_0 z$$

3) Do the math and find the polarizability:

$$\vec{p} = \varepsilon_0 \varepsilon_m \alpha \vec{E}_0$$

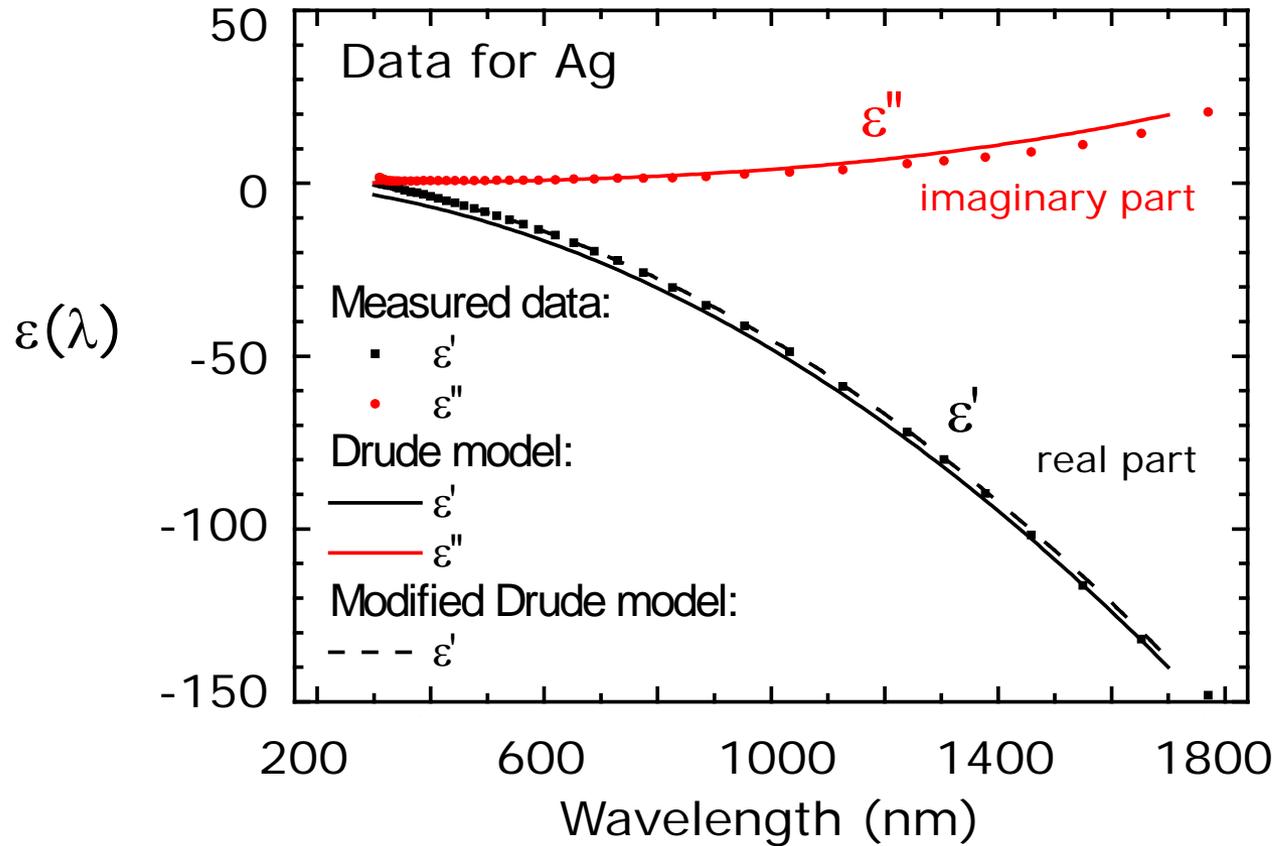
$$\alpha = 4\pi a^3 \left(\frac{\varepsilon - \varepsilon_m}{\varepsilon + 2\varepsilon_m} \right)$$



quasi-static approximation

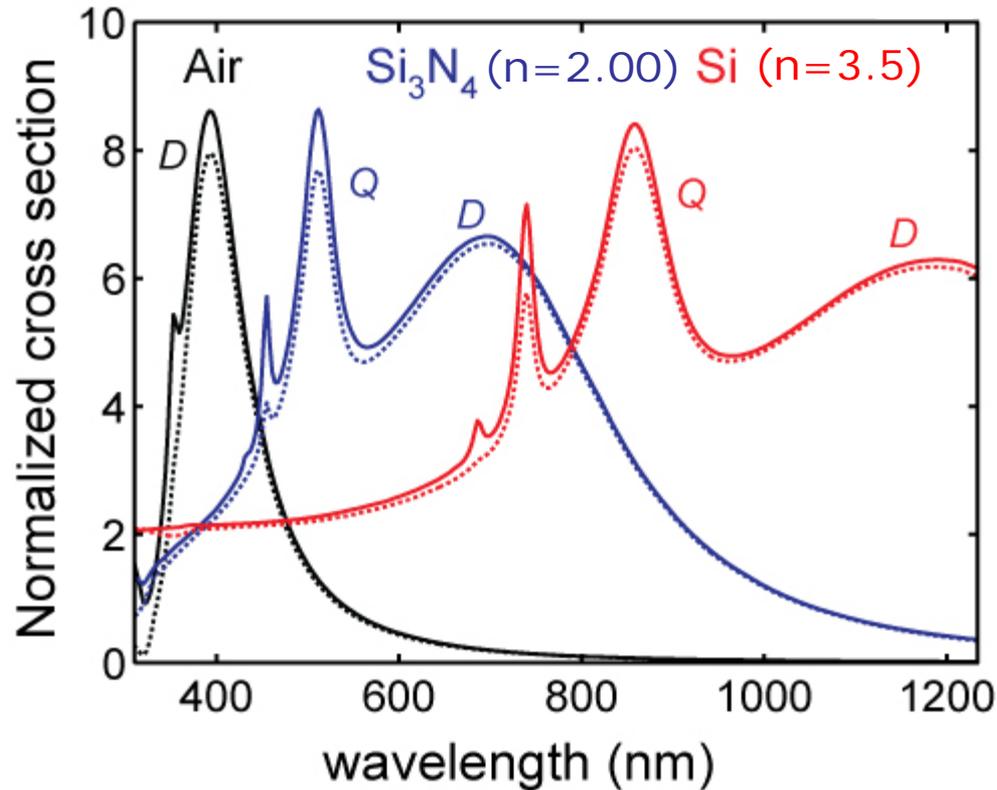
Plasmon resonance occurs if: $\varepsilon = -2\varepsilon_m$

Plasmon resonance of metal nanoparticles: quick tutorial (2/3)

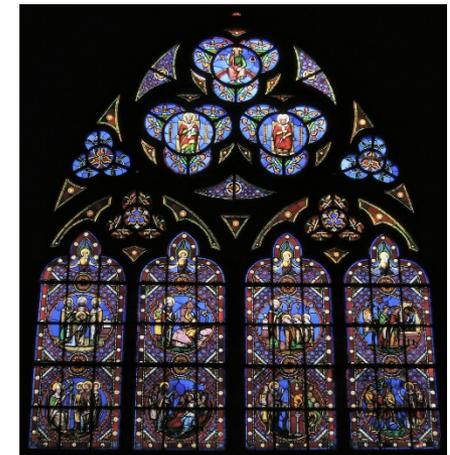


Plasmon resonance of metal nanoparticles: quick tutorial (3/3)

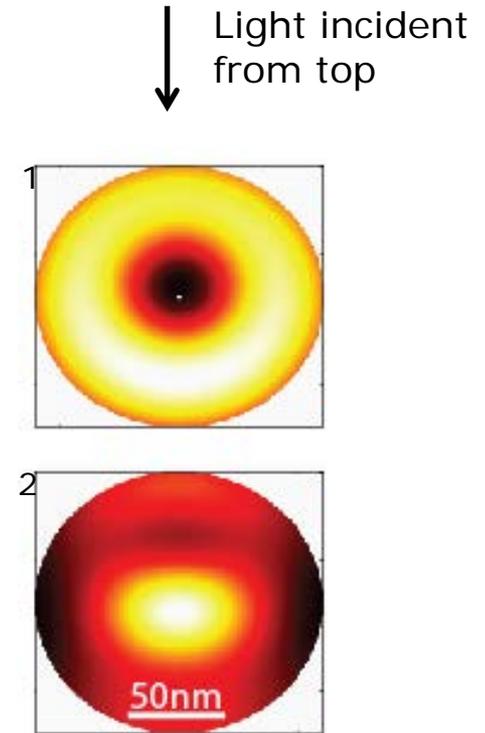
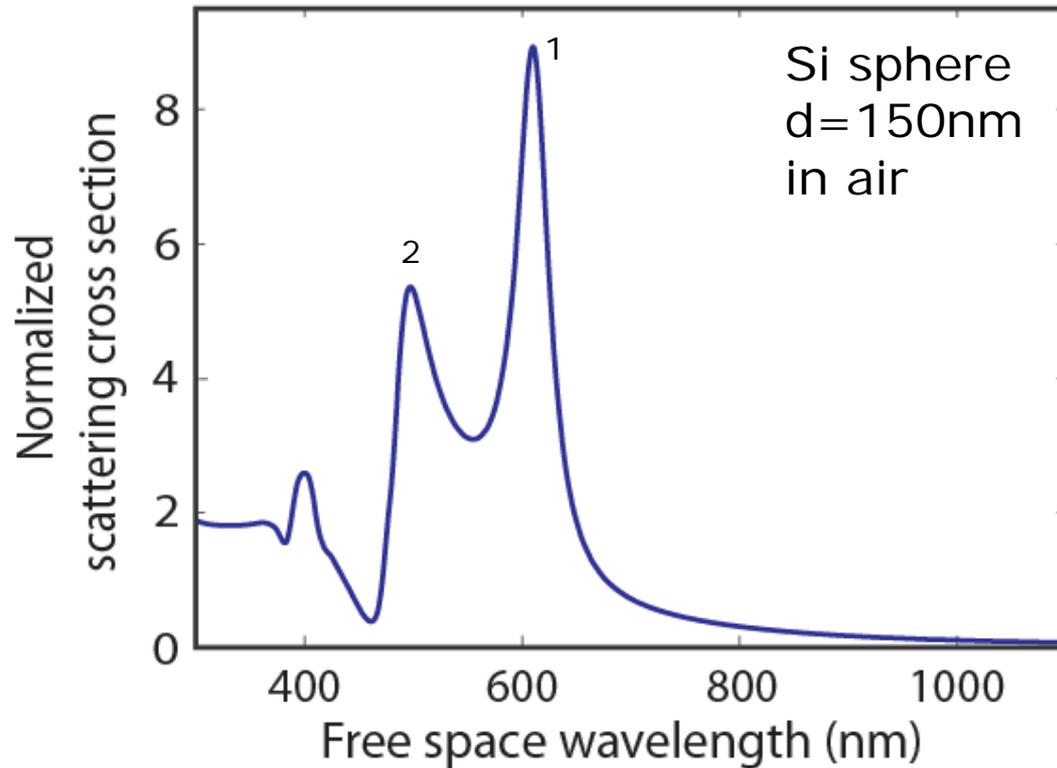
Plasmon resonance tunable by dielectric environment



Ag sphere
Diameter 100 nm



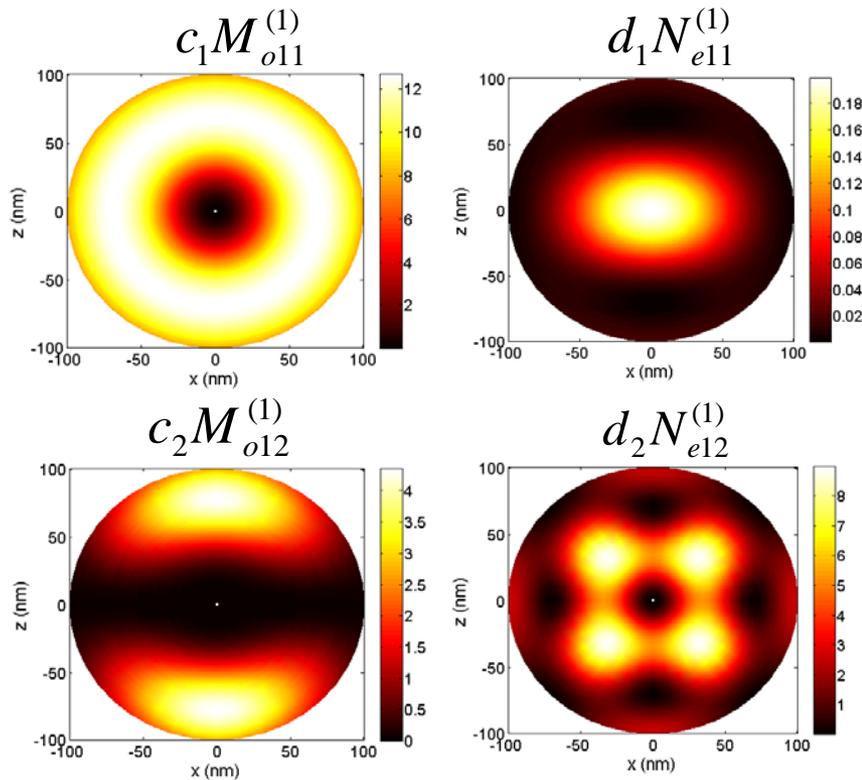
Mie resonance of dielectric nanoparticles: quick tutorial (1/2)



$$C_{scat} = \frac{2\pi}{k} \sum_{n=1}^{\infty} (2n+1) (|a_n|^2 + |b_n|^2)$$

Scattering cross section is sum of different modes

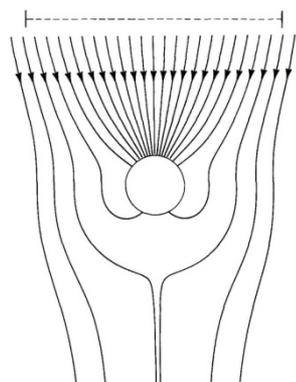
Mie resonance of dielectric nanoparticles: quick tutorial (2/2)



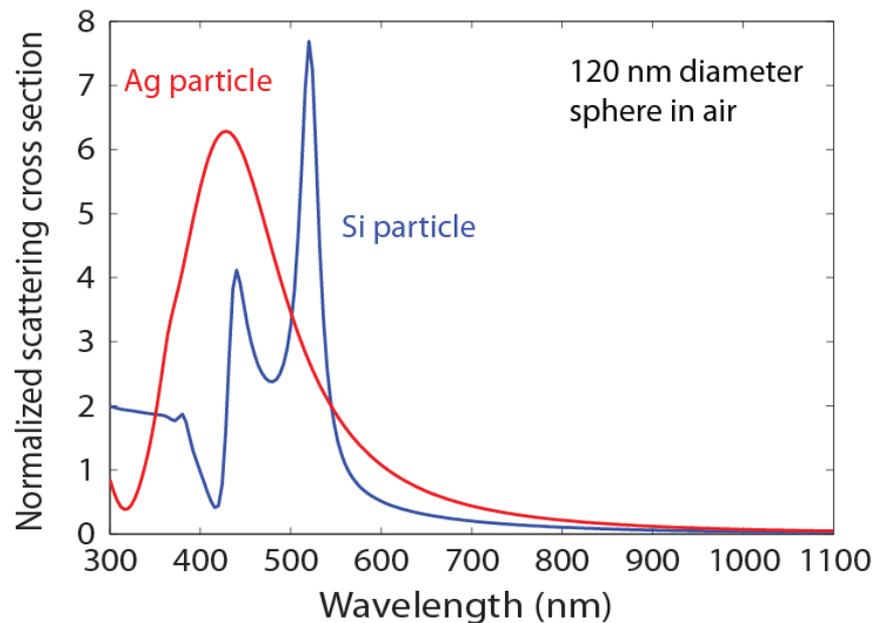
- Electric field inside particle is sum of different modes, each describing a precise geometrical pattern.
- Particle is a cavity for light

Comparing plasmon and Mie scattering

Mie theory: exact solution of Maxwell's equations for a sphere of any size embedded in homogeneous non-absorbing medium



$$Q_{scat} = \frac{C_{scat}}{C_{geom}}$$



- Optical cross section is larger than particle size
- Resonance quality factor $Q = 5-10$

Rayleigh scattering from small nanoparticles: quick tutorial (1/1)

Boundary conditions:

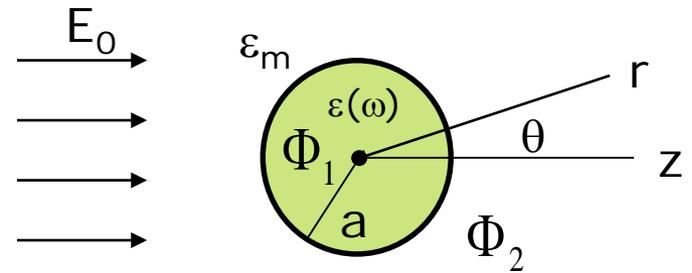
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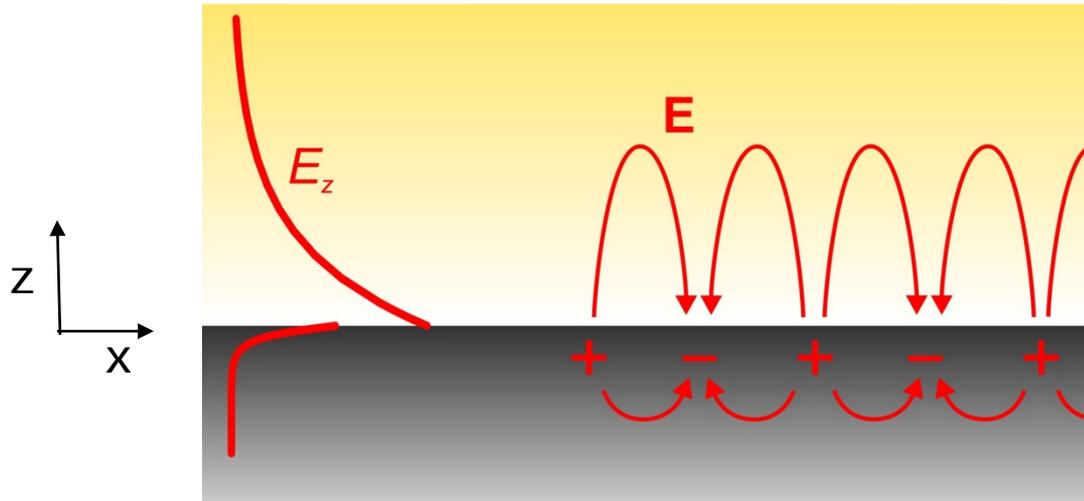


quasi-static approximation



Surface plasmon polaritons: quick tutorial (1/2)

SPP: EM wave at metal-dielectric interface



$$\vec{E}_d(x, z, t) = \vec{E}_{d,0} e^{i(k_x x + k_z z - \omega t)}$$

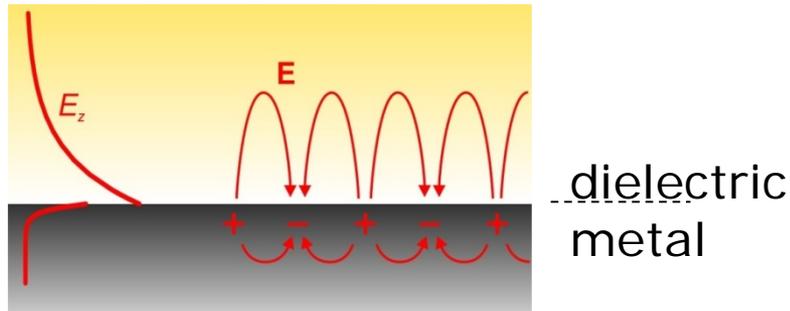
$$\vec{E}_m(x, z, t) = \vec{E}_{m,0} e^{i(k_x x - k_z z - \omega t)}$$

For propagating bound waves:

- k_x is real
- k_z is imaginary

EM wave is coupled to the **plasma oscillations** of the surface charges

Surface plasmon polaritons: quick tutorial (2/2)

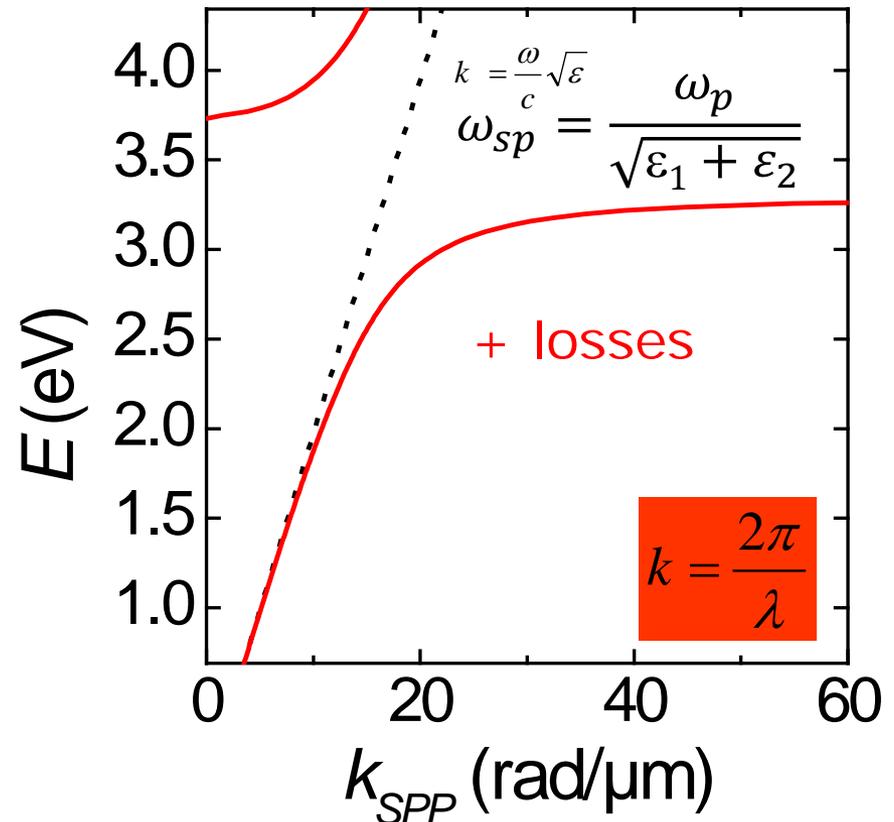


$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d} \right)^{1/2}$$

\downarrow \downarrow
 metal dielectric

$\epsilon(\omega)$ negative for metals

Dispersion of SPPs



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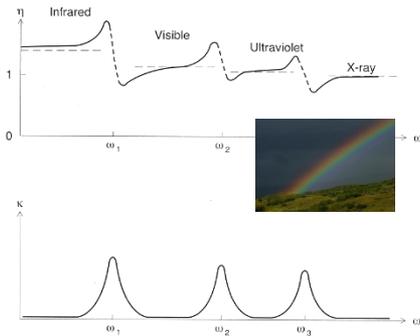
How to derive resonances and dispersion?

3) Dielectric scatterers

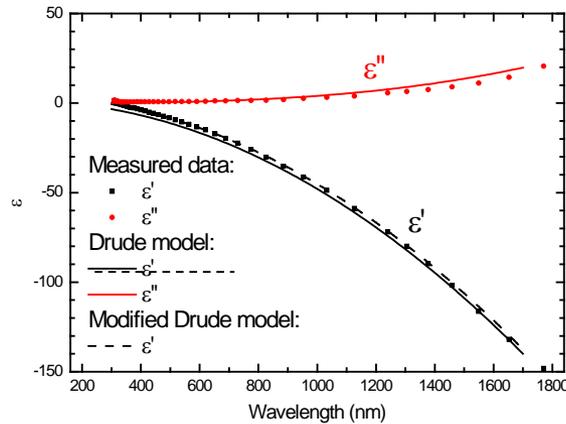
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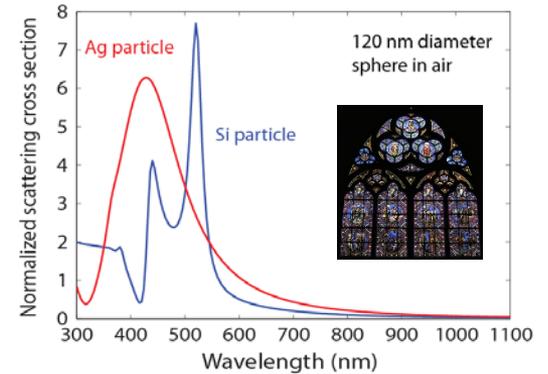
resonances and index



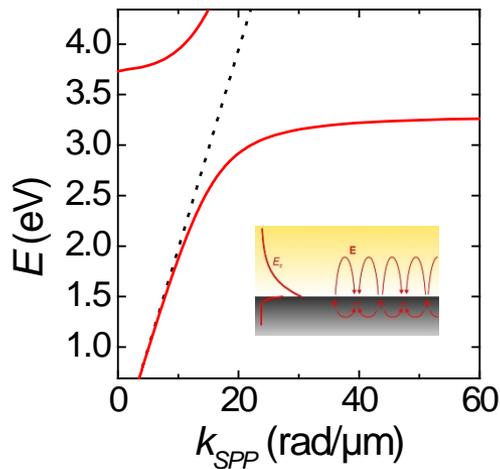
negative ω for metals



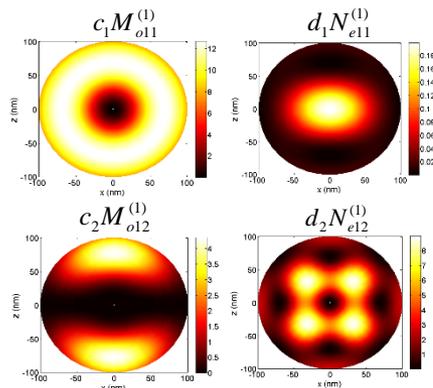
plasmon/Mie resonances



SP polariton dispersion



Mie modes in dielectrics



Rayleigh scattering

